QUESTION BANK

II PU MATHEMATICS

PUE

DEPARTMENT OF PRE UNIVERSITY EDUCATION

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CHAPTER -01

RELATIONS AND FUNCTIONS

One Mark questions.

1. Define a reflexive relation [K]

2. Define a symmetric relation . [K]

3. Define a transitive relation [K]

4. Define an equivalence relation [K]

5. A relation R on A = {1, 2, 3} defined by R = {(1, 1), (1, 2), (3, 3)} is not symmetric why ? [U]

6. Give an example of a relation which is symmetric but neither reflexive nor transitive. [U]

7. Give an example of a relation which is transitive but neither reflexive and nor symmetric. [U]

8. Give an example of a relation which is reflexive and symmetric but not transitive. [U]

9. Give an example of a relation which is symmetric and transitive but not reflexive . [U]

10. Define a one-one function. [K]

11. Define an onto function [K]

12. Define a bijective function. [K]

13. Prove that $f : R \to R$ defined by f(x) = x^2 is many-one. [U]

14. Prove that $f : Z \to Z$ defined by f(x) =1+ x^2 is not one one. [U]

15. Let A = {1, 2, 3} B = {4, 5, 6, 7} and f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. Show that f is one-one. [U]

16. Write the number of all one – one functions from the set A = {a, b, c} to itself. [U]

17. If A contains 3 elements and B contains 2 elements, then find the number of one-one functions from A to B. [U]

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18. Define a binary operation. [K]

19. Find the number of binary operations on the set {a,b}. [K]

20. On N, show that subtraction is not a binary operation. [U]

21. On Z^+ (The set of positive integers) define $*$ by $a*b=a-b$. Determine whether * is a binary operation or not. [U]

22. On Z^+, define $*$ by $a*b=ab$, (where Z^+ = The set of positive integers), Determine whether * is a binary operation or not. [U]

23. On Z^+, define $*$ by $a*b=ab^2$, (where Z^+ = The set of positive integers) Determine whether * is a binary operation or not. [U]

24. On Z^+, define $*$ by $a*b=a^b$, where Z^+ is the set of non negative integers, Determine whether * is a binary operation or not. [U]

25. On Z^+, define $*$ by $a*b=|a-b|$, (where Z^+ = The set of positive integers)Determine whether * is a binary operation or not. [U]

26. On Z^+, define $*$ by $a*b=a$, (where Z^+ = The set of positive integers),Determine whether * is a binary operation or not. [U]

27. Let * be a binary operation on N given by a * b = H.C.F. write the value of 22 * 4. [U]

28. Is * defined on the set A={1, 2, 3, 4, 5} by a * b = L C M of a and b, a binary operation ? Justify your answer. [U]

29. Let A = {1, 2, 3, 4, 5} and * is a binary operation on A defined by a * b = H C F of a and b Is * commutative? [U]

30. Let * be a b.o on N given by a * b = L.C.M.of a and b. find 5*7 [K]

31. Let A = {1, 2, 3, 4, 5} and * is a binary operation on A defined by a * b = H C F of a and b. Compute (2 * 3) . [U]

32. Let * be a b.o on N given by a * b = L.C.M.of a and b. find 20*16. [U]

33. On Z^+, define $*$ by $a*b=|a-b|$ where Z^+ is the set of non negative integers, determine whether * is a binary operation or not. [U]

34. On Z^+, define $*$ by $a*b=a^b$ where Z^+ is the set of non negative integers, determine whether * is a binary operation or not. [U]

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35. On Z^+(the set of nonnegative integers) define $*$ by $a*b = a-b \;\; \forall a,b \in Z^+$

Is * is a binary operation on Z^+ . [U]

36. On Z^+(the set of nonnegative integers) define $*$ by $a*b = |a-b| \;\; \forall a,b \in Z^+$.

Is * a binary operation on Z^+ . [U]

37. Show that ‘0’ is the identity for addition in R. [K]

38. Show that 1 is the identity for multiplication in R. [K]

39. Show that there is no identity element for subtraction (division) in R. [U]

40. On Q * is defined as , a * b = a – b .Find the identity if it exists. [U]

41. On Q * is defined as a * b = a + ab. Find the identity if it exists. [U]

42. On Q * is defined as $a*b = \frac{ab}{4} \;\; \forall\ a, b \in Q,$ find identity. [K]

43. On Q * is defined as $a*b = a+b \;\; \forall\ a,\ b \in N,$ find identity if it exists. [U]

44. On N, a * b = L.C.M of a and b. Find the identity of * in N. [K]

45. Show that –a is the inverse of a under addition in R. [K]

46. Show that $\frac{1}{a}$ is the inverse of a $(a \neq 0)$ under multiplication in R. [K]

47. Given a non-empty set X, consider the binary operation $*: P(X) \times P(X) \to P(X)$ given by $A*B = A \cap B \;\; \forall\ A, B \in P(X),$ where P(X) is the power set of X. Show that X is the identity element. [U]

48. Given a non-empty set X, let $*: P(X) \times P(X) \to P(X)$ defined by $A \times B = (A-B) \cup (B-A)$

Show that the empty set ϕ is the identity and all the elements of P(X). [U]

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Two Mark Questions.

1. Define a reflexive relation and give an example of it. [K]
2. Define a symmetric relation and give an example of it. [K]
3. Define a transitive relation and give an example of it. [K]
4. Define an equivalence relation and give an example of it. [K]
5. If $f : R \to R$ is defined by f(x) = 3x – 2. Show that f is one-one. [A]
6. If $f : N \to N$ given by $f(x) = x^2$ check whether f is one-one and onto. Justify your answer.[U]
7. If $f : Z \to Z$ given by $f(x) = x^2$ check whether f is one-one and onto. Justify your answer. [U]
8. If $f : R \to R$ given by $f(x) = x^2$ check whether f is one-one and onto. Justify your answer. **[U]**
9. If $f : N \to N$ given by $f(x) = x^3$ check whether f is one-one and onto. Justify your answer. [U]
10. If $f : Z \to Z$ given by $f(x) = x^3$ check whether f is one-one and onto. Justify your answer. [U]
11. Show that the function $f : N \to N$, given by f(x) = 2x is one-one but not onto. [U]
12. Show that the function given by f(1) = f(2) = 1 and f(x) = x – 1, for every x > 2, is onto but not one-one. [U]
13. Prove that the greatest integer function $f : R \to R$ given by f(x) = [x] is neither one-one nor onto [U]
14. Show that the modulus function $f : R \to R$ given by f(x) = | x | is neither one-one nor onto.[U]
15. Show that the Signum function $f : R \to R$ defined by $f(x) = \begin{cases} \frac{|x|}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$ is neither one-one nor onto [U]
16. Let $f : N \to N$ defined by $f(n) = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$ State whether f is bijective. Justify your answer. [U]

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17. Let A and B are two sets. Show that $f: A\times B \to B\times A$ such that f(a, b) = (b, a) is a bijective function. [U}

18. If $f: R \to R$ is defined by f(x) = 1 + x^2, then show that f is neither 1-1 nor onto. [U]

19. Prove that $f: R \to R$ given by f(x) = x^3 is onto. [U]

20. Let $f:\{2, 3, 4, 5\} \to \{3, 4, 5, 9\}$ and $g: \{3, 4, 5, 9\} \to \{7, 11, 15\}$ be functions defined f(2) = 3, f(3) = 4, f(4) = f(5) = 5 and g(3) = g(4) = 7 and g(5) = g(9) = 11. Find gof. [U]

21. Let $f: \{1, 3, 4\} \to \{1, 2, 5\}$ and $g:\{1, 2, 5\} \to \{1, 3\}$ given by f = {1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)} write down gof..**[U]**

22. If $f: A \to B$ and $g: B \to C$ are one-one then show that $gof: A \to C$ is also one-one.[K]

23. If $f: A \to B$ and $g: B \to C$ are onto then show that $gof: A \to C$ is also onto.[K]

24. State with reason whether $f: \{1, 2, 3, 4\} \to \{10\}$ with $f = \{(1, 10), (2, 10), (3, 10), (4,10)\}$ has inverse.[K]

25. State with reason whether $g = \{5, 6, 7, 8\} \to \{1, 2, 3, 4\}$ with $g = \{(5, 4), (6, 3), (7, 4), (8, 2)\}$ has inverse.[K]

26. State with reason whether $h: \{2, 3, 4, 5\} \to \{7, 9, 11, 13\}$ with $h = \{(2, 7), (3, 9), (4, 11), (5, 13)\}$ has inverse.[K]

27. Consider the binary operation V on the set {1, 2, 3, 4, 5} defined by a v b = min {a, b}. Write the operation table of the operation V. [K]

28. On Z, defined by a * b = a – b Determine whether * is commutative. [U]

29. On Q, defined by a * b = ab + 1. Determine whether * is commutative [U]

30. On Q , * defined by $a*b = \frac{ab}{2}$ Determine whether * is associative. [U]

31. On Z^+, * defined by $a*b = 2^{ab}$. Determine whether * associative. [U]

32. On R – {–1}, * defined by $a*b = \frac{a}{b+1}$ Determine whether * is commutative [U]

33. Verify whether the operation * defined on Q by $a*b = \frac{ab}{2}$ is associative or not . **[U]**

Three Mark Questions.

1) A relation R on the set A = {1, 2, 3......14} is defined as R = {(x, y) : 3x – y =0}. Determine whether R is reflexive, symmetric and transitive. [U]

2) A relation R in the set N of natural number defined as R = {(x, y) : y = x + 5 and x < 4}. Determine whether R is reflexive, symmetric and transitive. [U]

3) A relation 'R' is defined on the set A = {1, 2, 3, 4, 5} as R = {(x, y) : y is divisible by x}. Determine whether R is reflexive, symmetric, transitive.

4) Relation R in the set Z of all integers is defined as R = {(x, y) : x – y is an integer}. Determine whether R is reflexive, symmetric and transitive.

5) Determine whether R, in the set A of human beings in a town at a particular time is given by R = {(x, y) : x and y work at the same place}

6) Show that the relation R in **R,** the set of reals defined as R = {(a, b) : a $\leq$ b} is reflexive and transitive but not symmetric.

7) Show that the relation R on the set of real numbers R isdefined by R = {(a, b) : a $\leq$ b^2} is neither reflexive nor symmetric nor transitive.

8) Check whether the relation R in **R** the set of real numbers defined as R = {(a, b) : a $\leq$ b^3} is reflexive, symmetric and transitive.

9) Show the relation R in the set Z of integers give by R = {(a, b) : 2 divides (a – b)} is an equivalence relation.

10) Show the relation R in the set Z of integers give by R = {(a, b) : (a – b) is divisible by 2} is an equivalence relation.

11) Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a-b| is even} is an equivalence relation.

12) Show that the relation R on the set A of point on cordinate plane given by R = {(P, Q) distance OP = OQ, where O is origin is an equivalence relation.

13) Show that the relation R on the set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by R = {(a,b) $|a-b|$:is a multiple of 4 }is an equivalence relation.

14) Show that the relation R on the set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by R = {(a,b) :a=b }is an equivalence relation.

15) Show that the relation R on the set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by R = {(a,b) $|a-b|$:is a multiple of 4 }is an equivalence relation.

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16) Let T be the set of triangles with R – a relation in T given by R = $\{(T_1, T_2) : T_1$ is congruent to $T_2\}$

Show that R is an equivalence relation.

17) Let L be the set of all lines in a plane and R be the relation in L defined as R = $\{(L_1, L_2) : L_1$ is perpendicular to $L_2\}$. Show that R is symmetric but neither reflexive nor transitive.

18) Let L be the set of all lines in the XY plane and R is the relation on L by

R = $\{(l_1, l_2) : l_1$ is parallel to $l_2\}$. Show that R is an equivalence relation.

Find the set of all lines related to the line y = 2x + 4.

19) Show that the relation R defined in the set A of polygons as

R = $\{(P_1, P_2) : P_1$ and P_2 have same number of side } is an equivalence relation.

20) If R_1 and R_2 are two equivalence relations on a set, is $R_1 \cup R_2$ also an equivalence relation.? Justify your answer. [A]

21) If R_1 and R_2 are two equivalence relations on a set, then prove that $R_1 \cap R_2$ is also an equivalence relation.[A]

22) **F**ind gof and fog if $f : R \to R$ and $g : R \to R$ are given by f(x) = cos x and g(x) = $3x^2$.

Show that $gof \neq fog$. **[U]**

23) If f & g are functions from $R \to R$ defined by $f(x) = \sin x$ and $g(x) = x^2$ Show that

$gof \neq fog$. **[U]**

24) Find gof and fog, if $f(x) = |x|$ and $g(x) = |5x - 2|$ [U]

25) Find gof and fog, if $f(x) = 8x^3$ and $g(x) = x^{1/3}$ [U]

26) If $f : R \to R$ defined by f(x) = $(3 - x^3)^{1/3}$ then find fof(x). [U]

27) Consider $f : N \to N$, $g : N \to N$ and $h : N \to R$ defined as f(x) = 2x, g(y) = 3y + 4,

h(z) = sin z $\forall$ x, y, z $\in$ N. Show that $f \circ (g \circ h) = (f \circ g) \circ h$ [U]

28) Give examples of two functions f and g such that gof is one -one but g is not one-one.[S]

29) Give examples of two functions f and g such that gof is onto but f is not onto.[S]

Five Mark Questions

1) Let $A = R - \left\{\frac{7}{5}\right\}$, $B = R - \left\{\frac{3}{5}\right\}$ define $f : A \to B$ by $f(x) = \frac{3x+4}{5x-7}$ and

$g : B \to A$ by $g(x) = \frac{7x+4}{5x-3}$. Show that fog = I_B and gof = I_A. [U]

2) Consider $f : R \to R$ given by f(x) = 4x +3.Show that f is invertible.Find the inverse of f. [U]

3) Consider $f : R \to R$ given by f(x) = 10x + 7. Show tht f is invertible. Find the inverse of f. [U]

4) If $f(x) = \frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$, show that f o f(x) = x for all $x \neq \frac{2}{3}$. What is the inverse of f

[U]

5) Consider $f : R_+ \to [4, \infty)$ given by f(x) = $x^2 + 4$. Show that f is invertible with the inverse

[U]

f^{-1} of f given by $f^{-1}(y) = \sqrt{y-4}$, where R_+ is the set of all non-negative real numbers. [U]

6) Consider $f : R_+ \to [-5, \infty)$ given f(x) = $9x^2 + 6x - 5$. Show that f is invertible with

$f^{-1}(y) = \left(\frac{\sqrt{y+6}-1}{3}\right)$.

[U]

7) Let $f : N \to R$. be a function defined as f(x) = $4x^2 + 12x + 15$. Show that $f : N \to S$. Where S is the range of f, is invertible. Find the inverse of f

[U]

8) Let $Y = \{n^2 : n \in N\} \subset N$. Consider $f : N \to Y$ as f(n) = n^2. Show that f is invertible.

Find the inverse of f.

[U]

9) Show that $f : [-1, 1] \to R$, given by $f(x) = \frac{x}{x+2}$ is one-one.

Find the inverse of the function $f : [-1, 1] \to$ Range of f.

[U]

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10) Let $f : R - \left\{-\frac{4}{3}\right\} \to R$ be a function defined by define $f(x) = \frac{4x}{3x+4}$. Find the inverse of the function $f : R - \left\{-\frac{4}{3}\right\} \to$ Range of f.

[U]

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CHAPTER -02

INVERSE TRIGONOMETRIC FUNCTIONS

One Mark Questions

1. Write the domain of $f(x) = \sin^{-1}x$ [K]
2. Write the domain of $f(x) = \cos^{-1}x$ [K]
3. Write the range of $y = \cos^{-1}x$ [k]
4. Write the domain of $f(x) = \sec^{-1}x$ [K]
5. Write the principal value of branch of $f(x) = \sin^{-1}x$ [K]
6. Write the domain of $f(x) = \tan^{-1}x$ [K]
7. Write the set of all principal values of $\text{cosec}^{-1}x$. [K]
8. If $x = \sin^{-1}y$, then find set values of y . [K]
9. Write a range of $f(x) = \sin^{-1}x$ other than $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. [U]
10. Write a range of $f(x) = \cos^{-1}x$ other than $[0, \pi]$. [U]
11. Find $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ [K]
12. If $y = \cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$, then find value of y . [K]
13. Find the principal value $\sin^{-1}(-1)$.[K]
14. Find $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{-1}{2}\right)\right]$ [U]
15. Find $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$ [U]
16. Find $\sin\left[\frac{1}{2}\sin^{-1}(-1)\right]$ [U]
17. Find the principal value of $\tan^{-1}\left(-\sqrt{3}\right)$ [K]
18. Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ [K]

19. Find the principal value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$ [K]

20. Find the principal value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ [K]

21. Find the principal value of $\cos^{-1}\left(-1\right)$ [K]

22. Find the principal value of $\sec^{-1}(-2)$ [K]

23. Write the set of value of x for which $2\tan^{-1}x=\tan^{-1}\left[\frac{2x}{1-x^2}\right]$ holds. [K]

24. Write the set of value of x for which $2\tan^{-1}x=\cos^{-1}\left[\frac{1-x^2}{1+x^2}\right]$ holds [K]

25. Find the value of $\cos\left(\sec^{-1}x+\operatorname{cosec}^{-1}x\right)$ $|x|\geq 1$ [U]

26. Find the value of $\sin\left(\tan^{-1}a+\cot^{-1}a\right)$ [U]

27. Find $\sin(\tan^{-1}x)$,$|x|<1$. [U]

Two Mark Questions

1. Prove that $\cos^{-1}\left(-x\right)=\pi-\cos^{-1}x$, $x\in\left[-1,\ 1\right]$ [U]
2. Prove that $\sin^{-1}\left(-x\right)=-\sin^{-1}x$ where $x\in\left[-1,\ 1\right]$ [U]
3. Prove that $\tan^{-1}\left(-x\right)=-\tan^{-1}x$, $x\in R$ [U]
4. Prove that $\operatorname{cosec}^{-1}\left(-x\right)=-\operatorname{cosec}^{-1}x$, $|x|\geq 1$ [U]
5. Prove that $\sec^{-1}\left(-x\right)=\pi-\sec^{-1}x$, $|x|\geq 1$ [U]
6. Prove that $\cot^{-1}\left(-x\right)=\pi-\cot^{-1}x$, $x\in R$ U]
7. Find the value of $\tan^{-1}1+\cos^{-1}\left(\frac{-1}{2}\right)+\sin^{-1}\left(\frac{-1}{2}\right)$ [U]
8. Find the value of $\cos^{-1}\frac{1}{2}+2\sin^{-1}\left(\frac{1}{2}\right)$ [K]
9. Find the values of $\tan^{-1}\sqrt{3}-\sec^{-1}\left(-2\right)$ [K]
10. Find the value of $\tan^{-1}\sqrt{3}-\cot^{-1}\left(-\sqrt{3}\right)$ [K]
11. Find the value of $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$ [U]

12. Find the value of $\sin\left[\frac{\pi}{2}-\sin^{1}\left(-\frac{\sqrt{3}}{2}\right)\right]$ [U]

13. Find the value of $\sin\left[\frac{\pi}{2}-\sin^{1}\left(-\frac{\sqrt{3}}{2}\right)\right]$ [U]

14. Evaluate $\sin^{-1}\left[\sin 110^{0}\right]$ [K]

15. Evaluate $\sin^{-1}\left[\sin\frac{2\pi}{3}\right]$ [K]

16. Evaluate $\cos^{-1}\left[\cos 13\frac{\pi}{6}\right]$ [K]

17. Evaluate $\tan^{-1}\left[\tan\frac{7\pi}{6}\right]$ [K]

18. Prove that $\sin^{-1}x = \operatorname{cosec}^{-1}\frac{1}{x}$ [U]

19. Prove that $\cos^{-1}x = \sec^{-1}\frac{1}{x}$ [U]

20. Prove that $\tan^{-1}x = \cot^{-1}\frac{1}{x}$ [U]

21. Prove that $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}, \quad -1 \le x \le 1$ [U]

22. Prove that $\operatorname{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}, \quad |x| \ge 1$ [U]

23. Prove that $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}, \quad -\infty < x < \infty$ [U]

24. Evaluate $\sin^{-1}\left[\sin\frac{3\pi}{5}\right]$ [U]

25. Evaluate $\cos^{-1}\left[\sin\frac{\pi}{9}\right]$ [U]

26. If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$, then find x. [U]

27. Express $\tan^{-1}\left[\frac{1}{\sqrt{x^2-1}}\right], |x|>1,$ in the simplest form. [U]

28. Express $\tan^{-1}\left[\frac{x}{\sqrt{a^2-x^2}}\right], \ |x|<a$ in simplest form. [U]

29. Prove that $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\frac{1-x}{1+x}$, $x \in [0,1]$. [U]

30. Express $\tan^{-1}\left[\frac{3a^2x - x^3}{a^3 - 3ax^2}\right]$, $a > 0$, $\frac{-a}{\sqrt{3}} \le x \le \frac{a}{\sqrt{3}}$ in simplest form [U]

31. Write $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$, $0<x<\pi$ in simplest form [U]

32. Write $\tan^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right)$ $x \ne 2n\pi$ in simplest form [U]

33. Express $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$, $x>1$ in the simplest form [U]

34. Simplify $\tan^{-1}\left[\frac{3\cos x - 4\sin x}{4\cos x + 3\sin x}\right]$, if $\frac{3}{4}\tan x > -1$. [U]

35. Prove that $\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right) = \tan^{-1}\frac{a}{b} - x$ [U]

36. Prove that $2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, $|x| \le 1$ [U]

37. Prove that $2\tan^{-1}x = \cos^{-1}\frac{1-x^2}{1+x^2}$, $x \ge 0$ [U]

38. Prove that $2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}$ $\quad -1 < x < 1$ [U]

39. Prove that $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\sin^{-1}x$, $\frac{-1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$ [U]

40. Prove that $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\cos^{-1}x$, $\frac{1}{\sqrt{2}} \le x \le 1$ [U]

41. Prove that $3\sin^{-1}x = \sin^{-1}\left(3x - 4x^3\right)$, $x \in \left[\frac{-1}{2}, \frac{1}{2}\right]$ [U]

42. Prove that $3\cos^{-1}x = \cos^{-1}\left(4x^3 - 3x\right)$, $x \in \left[\frac{1}{2}, 1\right]$ [U]

43. Prove that $\sin\left(\tan^{-1}x\right) = \frac{x}{\sqrt{1+x^2}}$, $|x| < 1$ [U]

44. Prove that $\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$ [U]

45. Prove that $2\tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\frac{3}{4}$ [U]

46. Prove that $2\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{24}{7}$. [U]

47. Prove that $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{2}{11}=\tan^{-1}\frac{3}{4}$ [U]

48. Prove that $\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24}=\tan^{-1}\frac{1}{2}$ [U]

49. Simplify $\tan^{-1}\left[\frac{a\cos x-b\sin x}{b\cos x+a\sin x}\right]$, if $\frac{a}{b}\tan x>-1.$ **[U]**

Three Mark Questions

1. Prove that $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$, $xy<1$ [U]

2. Prove that $\tan^{-1}x-\tan^{-1}y=\tan^{-1}\left(\frac{x-y}{1+xy}\right)$, $xy>-1$ [U]

3. Prove that $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$ [U]

4. Prove that $\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$ [U]

5. Prove that $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)=\frac{\pi}{4}$ [U]

6. Write $\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$, $0<x<\pi$ in simplest form [U]

7. Prove that $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,\ \frac{-1}{2}\le x\le 1$ [U]

8. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2},\ x\in\left(0\ \ \frac{\pi}{4}\right)$ [U]

9. Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, $-\frac{\pi}{2}<x<\frac{3\pi}{2}$ in the simplest form [U]

10. Prove that $\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{77}{36}$ [U]

11. Prove that $\sin^{-1}\frac{3}{5}-\sin^{-1}\frac{8}{17}=\cos^{-1}\frac{84}{85}$ [U]

12. Prove that $\tan^{-1}\frac{63}{16}=\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}$ [U]

13. Prove that $\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}=\cos^{-1}\frac{33}{65}$ [U]

14. Prove that $\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{56}{65}$: [U]

15. Prove that $\sin^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=\pi$ [U]

16. Prove that $\tan^{-1}x+\tan^{-1}\frac{2x}{1-x^2}=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ $|x|<\frac{1}{\sqrt{3}}$ [U]

17. Prove that $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{2}{11}+\tan^{-1}\left(\frac{4}{3}\right)=\frac{\pi}{2}$ [U]

18. Solve for x $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$: [U]

19. Solve for x : $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$ [U]

20. Solve : $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$ [U]

21. Solve : $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x$ $(x>0)$ [U]

22. Solve : $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$ [U]

CHAPTER -03

MATRICES

One mark questions:

1. Define a scalar matrix. **(K)**
2. Define an Identity matrix. **(K)**
3. Define a diagonal matrix. **(K)**

4. In the matrix $A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$, write: (i) the order of the matrix

(ii) The number of elements (iii) Write the elements a_{13}, a_{21}, a_{33}, a_{24}, a_{23} . **(U)**

5. If a matrix has 24 elements, what are the possible orders it can have? What, if it has 13 elements? **(U)**

6. If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements? **(U)**

7. Find the number of all possible matrices of order 3 ×3 with each entry 0 or 1 ? **(U)**

8. If a matrix has 8 elements, what are the possible orders it can have? **(U)**

9. Construct a 2 × 2 matrix, A = $[a_{ij}]$, whose elements are given by; **(A)**

(i) $a_{ij} = \frac{(i+j)^2}{2}$ (ii) $a_{ij} = \frac{i}{j}$ (iii) $a_{ij} = \frac{(i+2j)^2}{2}$

10. Construct a 3 × 3 matrix whose elements are given by $a_{ij} = \frac{1}{2}|i-3j|$ **(A)**

11. Construct a 3 × 4 matrix, whose elements are given by: **(A)**

(i) $a_{ij} = \frac{1}{2}|-3j+j|$ (ii) $a_{ij} = 2i - j$

12. Find the values of x, y and z from the following equations:

(i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$ **(U)** (ii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$ **(A)**

13. Find x and y, if $2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$ **(U)**

14. If $x\begin{bmatrix}2\\ 3\end{bmatrix}+y\begin{bmatrix}-1\\ 1\end{bmatrix}=\begin{bmatrix}10\\ 5\end{bmatrix}$, find the values of x and y. **(U)**

15. Find X, if $Y=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$ and $2X+Y=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}$ **(A)**

16. Find the values of x and y from the following equation

$$2\begin{bmatrix}x & 5\\ 7 & y-3\end{bmatrix}+\begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix}=\begin{bmatrix}7 & 6\\ 15 & 16\end{bmatrix}$$ **(U)**

17. Find the value of a, b, c and d from the equation:

$$\begin{bmatrix}a-b & 2a+c\\ 2a-b & 3c+d\end{bmatrix}=\begin{bmatrix}-1 & 5\\ 0 & 13\end{bmatrix}$$ **(A)**

18. Show that the matrix $A=\begin{bmatrix}1 & -1 & 5\\ -1 & 2 & 1\\ 5 & 1 & 3\end{bmatrix}$ is a symmetric matrix. **(U)**

19. Show that the matrix $A=\begin{bmatrix}0 & 1 & -1\\ -1 & 0 & 1\\ 1 & -1 & 0\end{bmatrix}$ is a skew symmetric matrix. **(U)**

20. Let A = $\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}$, B = $\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}$, C= $\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix}$. Find each of the following

(i) A + B (ii) A – B (iii) 3A – C (iv) AB (v) BA **(U)**

21. Consider the following information regarding the number of men and women workers in three factories I, II and III

	Men Workers	Women workers
I	30	25
II	25	31
III	27	26

Represent the above information in the form of a 3 × 2 matrix. What does the entry in the third row and second column represent? **(U)**

22. Given $A=\begin{bmatrix}\sqrt{3} & 1 & 1\\ 2 & 3 & 0\end{bmatrix}$ and $B=\begin{bmatrix}2 & \sqrt{5} & 1\\ -2 & 3 & \frac{1}{2}\end{bmatrix}$, find A + B **(U)**

23. If A = $\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{bmatrix}$ and B = $\begin{bmatrix}3 & -1 & 3\\ -1 & 0 & 2\end{bmatrix}$, then find 2A – B **(U)**

24. Find AB, if A = $\begin{bmatrix}6 & 9\\ 2 & 3\end{bmatrix}$ and B = $\begin{bmatrix}2 & 6 & 0\\ 7 & 9 & 8\end{bmatrix}$ **(U)**

25. If A = $\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$ and B = $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$, then prove that i) AB = $\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$ and ii)BA = $\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$. **(U)**

26. Find AB, if A = $\begin{bmatrix}0 & -1\\ 0 & 2\end{bmatrix}$ and B = $\begin{bmatrix}3 & 5\\ 0 & 0\end{bmatrix}$ **(U)**

27. Simplify $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$ **(A)**

28. Find P⁻¹, if it exists, given P = $\begin{bmatrix}10 & -2\\ -5 & 1\end{bmatrix}$ **(A)**

29. Find the transpose of each of the following matrices: A = $\begin{bmatrix}3 & \sqrt{3} & 2\\ 4 & 2 & 0\end{bmatrix}$ and B = $\begin{bmatrix}2 & -1 & 2\\ 1 & 2 & 4\end{bmatrix}$.

(U)

30. If A = $\begin{bmatrix}3 & \sqrt{3} & 2\\ 4 & 2 & 0\end{bmatrix}$ and B = $\begin{bmatrix}2 & -1 & 2\\ 1 & 2 & 4\end{bmatrix}$, verify that **(A)**

(i) $(A')' = A$ (ii) $(A+B)' = A'+B'$

31. Compute the following **(A)**

(i) $\begin{bmatrix}a & b\\ -b & a\end{bmatrix}+\begin{bmatrix}a & b\\ b & a\end{bmatrix}$ (ii) $\begin{bmatrix}a^2+b^2 & b^2+c^2\\ a^2+c^2 & a^2+b^2\end{bmatrix}+\begin{bmatrix}2ab & 2bc\\ -2ac & -2ab\end{bmatrix}$

(iii) $\begin{bmatrix}-1 & 4 & -6\\ 8 & 5 & 16\\ 2 & 8 & 5\end{bmatrix}+\begin{bmatrix}12 & 7 & 6\\ 8 & 0 & 5\\ 3 & 2 & 4\end{bmatrix}$ (iv) $\begin{bmatrix}\cos^2 x & \sin^2 x\\ \sin^2 x & \cos^2 x\end{bmatrix}+\begin{bmatrix}\sin^2 x & \cos^2 x\\ \cos^2 x & \sin^2 x\end{bmatrix}$

32. Compute the indicated products: **(U)**

(i) $\begin{bmatrix} a & b \\ -b & 1 \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ (ii) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

(iv) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ (v) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$ (vi) $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$

Three marks questions:

1. Find the values of x, y and z from the following equations: $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$. **(S)**

2. Solve the equation for x, y, z and t, if $2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$ **(U)**

3. Given $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$, find the values of x, y, z and w. **(S)**

4. Find the values of a, b, c and d from the following equation

$$\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$$ **(A)**

5. If $\begin{bmatrix} x+3 & z+4 & 2y-7 \\ -6 & a-1 & 0 \\ b-3 & -21 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ -6 & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$ **(S)**

Find the values of a, b, c, x, y and z.

6. Using elementary transformations, find the inverse of each of the matrices **(A)**

(i) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ iii. $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$ iv. $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$ v. $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$

vi. $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ vii. $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$ viii. $\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$ ix. $\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$ xi. $\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$

xii. $\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$ xiii. $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ xiv. $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$

7. By using elementary operations, find the inverse of the matrix A = $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ **(U)**

8. Find X and Y, if X + Y = $\begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and X – Y = $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ **(U)**

9. Find X and Y, if 2X + 3Y = $\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and 3X + 2Y = $\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$ **(U)**

10. If A = $\begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$, then find the matrix X, such that 2A + 3X = 5B. **(A)**

11. Find X and Y, if X + Y = $\begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$ and X – Y = $\begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$ **(U)**

12. If A = $\begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix}$ and B = $\begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$, then compute 3A – 5B **(A)**

13. If F(x) = $\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x)\cdot F(y) = F(x+y)$ **(A)**

14. If (i) A = $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then verify that A′ A = I **(A)**

(ii) If A = $\begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$, then verify that A′ A = I

15. Show that $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$ **(U)**

16. Show that $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ **(A)**

17. If A = $\begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, then verify that

(i) $(A + B)' = A' + B'$ (ii) $(A - B)' = A' - B'$ **(U)**

18. If A′ = $\begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then verify that **(U)**

(i) $(A + B)' = A' + B'$ (ii) $(A - B)' = A' - B'$

19. If A′ = $\begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A + 2B)'$ **(U)**

20. If A = $\begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$ then find AB, BA. Show that $AB \neq BA$. **(U)**

21. In a legislative assembly election, a political group hired a public relations firm to promote its candidate in three ways: telephone, house calls, and letters. The cost per contact (in paise) is given in matrix A as

Cost per contact

$$A = \begin{array}{c} \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix} \end{array} \begin{array}{l} Telephone \\ Hou\sec all \\ Letter \end{array}$$

The number of contacts of each type made in two cities X and Y is given by

$$\begin{array}{cc} & \begin{array}{ccc} Telephone & Hou\sec all & Letter \end{array} \\ B = & \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \end{array} \begin{array}{l} \to X \\ \to Y \end{array}$$

. Find the total amount spent by the group in the two cities X and Y. **(A)**

22. A trust fund has RS. 30,000 that must be invested in two different types of bonds. The first bond pays5 % interest per year, and the second bond pays 7 % interest per year. Using matrix multiplication, determine how to divide Rs. 30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of :

(a) Rs. 1800 (b) Rs. 2000 **(A)**

23. A book shop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs. 80, Rs. 60 and Rs. 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra. **(A)**

24. If A and B are symmetric matrices of the same order, then show that AB is symmetric if and only if A and B commute, that is AB = BA. **(K)**

25. If A = $\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$ **(A)**

26. For the matrix A = $\begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that **(U)**

(i) (A + A′) is a symmetric matrix (ii) (A - A′) is a skew symmetric matrix

27. Find $\frac{1}{2}(A+A')$ and $\frac{1}{2}(A-A')$, when A = $\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ **(U)**

28. Express the following matrices as the sum of a symmetric and skew symmetric matrix: **(U)**

(i) $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ (ii) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

29. Express the matrix B = $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. **(U)**

30. If A and B are symmetric matrices of the same order, then show that AB is symmetric if and only if AB = BA. **(K)**

31. If A and B are invertible matrices of the same order, then prove that $(AB)^{-1} = B^{-1}A^{-1}$ **(K)**

32. Prove that for any square matrix A with real number entries, $A+A'$ is a symmetric matrix and $A-A'$ is a skew symmetric matrix. **(K)**

33. Prove that any square matrix can be expressed as the sum of symmetric and skew symmetric matrix. **(K)**

34. Prove inverse of a square matrix, if it exist, is unique. **(K)**

Five marks questions:

1. If $A=\begin{bmatrix}-2\\4\\5\end{bmatrix}, B=\begin{bmatrix}1 & 3 & -6\end{bmatrix}$, verify that $(AB)' = B'A'$. **(U)**

4. If A = $\begin{bmatrix}1 & 2 & -3\\5 & 0 & 2\\1 & -1 & 1\end{bmatrix}$, B = $\begin{bmatrix}3 & -1 & 2\\4 & 2 & 5\\2 & 0 & 3\end{bmatrix}$ and C = $\begin{bmatrix}4 & 1 & 2\\0 & 3 & 2\\1 & -2 & 3\end{bmatrix}$ **(U)**

Then compute (A + B) and (B – C).Also, verify that A + (B – C) = (A + B) – C

5. If A = $\begin{bmatrix}1 & 1 & -1\\2 & 0 & 3\\3 & -1 & 2\end{bmatrix}$, B = $\begin{bmatrix}1 & 3\\0 & 2\\-1 & 4\end{bmatrix}$ and C = $\begin{bmatrix}1 & 2 & 3 & -4\\2 & 0 & -2 & 1\end{bmatrix}$, find A(BC), (AB)C and show that (AB)C = A(BC). **(U)**

6. Find $A^2-5A+6I$, if $A=\begin{bmatrix}2 & 0 & 1\\2 & 1 & 3\\1 & -1 & 0\end{bmatrix}$ **(U)**

7. If $A=\begin{bmatrix}0 & 6 & 7\\-6 & 0 & 8\\7 & -8 & 0\end{bmatrix}, B=\begin{bmatrix}0 & 1 & 1\\1 & 0 & 2\\1 & 2 & 0\end{bmatrix}, C=\begin{bmatrix}2\\-2\\3\end{bmatrix}$

Calculate AC, BC and (A + B) C. Also, verify that (A + B)C = AC + BC. **(U)**

8. If $A=\begin{bmatrix}1 & 1 & 1\\1 & 2 & -3\\2 & -1 & 3\end{bmatrix}$, verify that $A^3-6A^2+5A+11I=O$,

where O is zero matrix of order 3 x 3. **(A)**

9. If A = $A=\begin{bmatrix}1 & 2 & 3\\3 & -2 & 1\\4 & 2 & 1\end{bmatrix}$, then show that $A^3-23A-40I=O$. **(A)**

10. If $A=\begin{bmatrix}1 & 0 & 2\\0 & 2 & 1\\2 & 0 & 3\end{bmatrix}$, prove that $A^3-6A^2+7A+2I=0$. **(A)**

11. Let A = $A=\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, B=\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}, C=\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$. Find a matrix D such that $CD-AB=O$ **(U)**

12. If $A=\begin{bmatrix} 0 & -\tan\dfrac{\alpha}{2} \\ \tan\dfrac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that

$I+A=(I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$. **(S)**

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PUE

CHAPTER -4

DETERMINANTS

One marks questions;

1. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find$|2A|$.
2. Find the values of x for which $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$.
3. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, Find $|2A|$.
4. If A is a square matrix with $|A| = 6$. Find the value of $|AA'|$.
5. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find$|A^{-1}|$.
6. If A is a square matrix of order 3 and $|A| = 4$. Find the value of $|2A|$.
7. If A is a square matrix and $|A| = 2$, then find the value of $|AA'|$.
8. If A is a invertible matrix of order 2, then find $|A^{-1}|$.
9. If A is a square matrix of order 3 and $|A| = 4$, then find$|adjA|$.
10. Find x if $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$.
11. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, find $A \cdot adjA$.
12. Without expansion find the value of $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$.
13. Evaluate $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$.
14. Without expanding, prove that $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$. (2 Marks Question)
15. Evaluate $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$.
16. If $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$, find x.
17. If A is a square matrix of order 3×3, find the value of $|KA|$.
18. If $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$, find the values of x.
19. Evaluate $\begin{vmatrix} \sin 30° & \cos 30° \\ -\sin 60° & \cos 60° \end{vmatrix}$.
20. If $A = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then find $|3A|$.
21. Examine the consistency of the system of linear equations $x + 2y = 2$ and $2x + 3y = 3$.
22. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that $|2A| = 4|A|$.
23. Evaluate $\begin{vmatrix} 0 & \sin\alpha & -\cos\alpha \\ -\sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{vmatrix}$.

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24. If $A=\begin{bmatrix}1 & 1 & -2\\ 2 & 1 & -3\\ 5 & 4 & -9\end{bmatrix}$, find $|A|$.

25. Evaluate $\begin{vmatrix}x^2-x+1 & x-1\\ x+1 & x+1\end{vmatrix}$.

Two marks questions;

1. Find the equation of the line joining the points $(3,1)$ and $(9,3)$ using determinants.
2. Find the equation of the line joining the points $(1,2)$ and $(3,6)$ using determinants.
3. If each element of a row is expressed as the sum of two elements then verify for a third order determinant that the determinant can be expressed as sum of two determinants.
4. Find the area of the triangle whose vertices are $(3,8),(-4,2)$and $(5,1)$ using determinants.
5. Prove that the value of the determinant remains unaltered if its rows and columns are interchanged by considering a third order determinant.
6. Without expansion, prove that $\begin{vmatrix}2 & 7 & 65\\ 3 & 8 & 75\\ 5 & 9 & 86\end{vmatrix}=0$.
7. If the area of the triangle with vertices $(-2,0)(0,4)$ and $(0,k)$ is 4 square units. Find the value of k using determinants.
8. Examine the consistency of the system of equations $x+3y=5$ and $2x+6y=8$.
9. Without expansion find the value of $\begin{vmatrix}4 & a & b+c\\ 4 & b & c+a\\ 4 & c & a+b\end{vmatrix}$.
10. Find the area of the triangle whose vertices are $(1,0)(6,0)$and $(4,3)$ using determinants.
11. Find$k$, if the area of the triangle is 3 square units and whose vertices are $(k,0)(1,3)$ and $(0,0)$ using determinants.
12. If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, show that $A^2-5A+7I=0$ and hence find A^{-1}. (4 Marks Question)
13. Prove that $|adjA|=|A|^2$, where A is the matrix of order 3×3.
14. Find the area of the triangle whose vertices are $(3,8),(-4,2)$ and $(5,1)$ using determinants.
15. Find the area of the triangle whose vertices are $(2,7),(1,1)$ and (10,8) using determinants.
16. Without expansion, prove that $\begin{vmatrix}x & a & x+a\\ y & b & y+b\\ z & c & z+c\end{vmatrix}=0$.
17. If in a determinant, any two rows or columns are interchanged, then prove that the sign of the determinant changes.
18. Prove that $\begin{vmatrix}x+4 & 2x & 2x\\ 2x & x+4 & 2x\\ 2x & 2x & x+4\end{vmatrix}=(5x+4)(4-x)^2$. (4 Marks Question)
19. If each element of a row or a column of a determinant is multiplied by a constant k, then prove that the whole determinant is multiplied by the same constant k.
20. Solve the system of linear equations using matrix method:
    (i) $2x+5y=1,\ 3x+2y=7$ (ii) $5x+2y=4,\ 7x+3y=5$
    (iii) $5x+2y=3,\ 3x+2y=5$ (iv) $4x-3y=3,\ 3x-5y=7$.
21. Prove that $\begin{vmatrix}y+k & y & y\\ y & y+k & y\\ y & y & y+k\end{vmatrix}=k^2(3y+4)$. (4 Marks Question)

PUE

Four marks questions;

1. Prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc.$
2. Prove that $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3.$
3. Prove that $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx).$
4. Prove that $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2.$
5. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c).$
6. Prove that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3.$
7. Show that $\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca).$
8. Prove that $\begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix} = (1-x^2)\begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}.$
9. Prove that $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3.$
10. Prove that $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+2b & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2.$
11. Prove that $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}.$
12. Prove that $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}.$
13. If x, y, z are all different from zero and $\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then show that $1 + xyz = 0$.
14. If $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = 0$ and x, y, z are all different from zero

    Then prove that $1 + \sum\left(\frac{1}{x}\right) = 0.$
15. Show that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right).$
16. Prove that $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = (a-b)(b-c)(c-a).$

17. Prove that $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$.

18. Prove that $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4\,a^2b^2c^2$.

19. Prove that $\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$.

20. Prove that $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$.

21. Prove that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$.

22. Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$.

23. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$.

24. Prove that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ.

25. Evaluate $\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$.

26. Prove that $\begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) \end{vmatrix} = 0$.

27. Prove that $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1$.

28. Solve the equation $\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0, a \neq 0$.

Five marks questions;

1. Solve the system of equations $x+y+z=6$, $y+3z=11$ and $x-2y+z=0$ by matrix method.
2. Solve the system of equations $3x-2y+3z=8$, $2x+y-z=1$ and $4x-3y+2z=4$ by matrix method.
3. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the system of equations $2x-3y+5z=11, 3x+2y-4z=-5$ and $x+y-2z=-3$.
4. The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs 60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is Rs 90. The cost of 6 kg onion, 2 kg wheat and 3 kg rice is Rs 70. Find the cost of each item per kg by matrix method.

5. The sum of three numbers is 6. If we multiply the third number by 3 and add the second number to it we get 11. By adding the first and third numbers, we get double the second number. Represent it algebraically and find the numbers using matrix method.
6. Solve the equations $2x + y + z = 1, x - 2y - z = \frac{3}{2}$ and $3y - 5z = 9$ by matrix method.
7. Solve the equations $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4,\ \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$ and $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$ by matrix method.
8. Use the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations $x - y + 2z = 1,\ 2y - 3z = 1, 3x - 2y + 4z = 2$.
9. Solve the system of equations $x - y + 2z = 7, 3x + 4y - 5z = -5$ and $2x - y + 3z = 12$ by matrix method.
10. Solve the system of equations $x - y + z = 4,\ 2x + y - 3z = 0$ and $x + y + z = 2$ by matrix method.
11. Solve the system of equations $2x + 2y + 3z = 4,\ x - 2y + z = -4$ and $3x - 4y - 2z = 3$ by matrix method.

PUE

CHAPTER-5

CONTINUITY AND DIFFERENTIABILITY

CONTINUITY

TWO MARK QUESTIONS

1. Check the continuity of the function f given by $f(x)=2x+3$ at $x=1$. (U)

2. Examine whether the function f given by $f(x)=x^2$ is continuous at $x=0$. (U)

3. Discuss the continuity of the function f given by $f(x)=|x|$ at $x=0$. (U)

4. Show that the function f given by $f(x) = \begin{cases} x^3+3, & \text{for } x \neq 0 \\ 1, & \text{for } x = 0 \end{cases}$ is not continuous at $x=0$.(U)

5. Check the points where the constant function $f(x)=\mathrm{k}$ is continuous. (U)

6. Prove that the identity function on real numbers given by $f(x)=x$ is continuous at every real number. (U)

7. Is the function defined by $f(x)=|x|$, a continuous function? (U)

8. Discuss the continuity of the function f given by $f(x)=x^3+x^2-1$. (U)

9. Discuss the continuity of the function f defined by $f(x)=\frac{1}{x}$, $x \neq 0$. (U)

10. Show that every polynomial function is continuous. (U)

11. Show that every rational function is continuous. (U)

12. Prove that the function $f(x)=5x-3$ is, continuous at $x=0$.(U)

13. Prove that the function $f(x)=5x-3$ is, continuous at $x=-3$.(U)

14. Prove that the function $f(x)=5x-3$ is, continuous at $x=5$.(U)

15. Examine the continuity of the function $f(x)=2x^2-1$ at $x=3$.(U)

16. Examine the following functions for continuity: **(Each question of 2 Marks)**

a) $f(x)=x-5$ **b)** $f(x)=|x-5|$

c) $f(x)=\dfrac{x^2-25}{x+5}$, $x \neq -5$ **d)** $f(x)=\dfrac{1}{x-5}$, $x \neq 5$. (U)

17. Prove that the function $f(x)=x^n$ is continuous at $x=n$, where n is a positive integer. (U)

18. Discuss the continuity of the following functions: **(Each question is of 2 Marks)**

a) $f(x)=\sin x+\cos x$ **b)** $f(x)=\sin x-\cos x$ **c)** $f(x)=\sin x \cdot \cos x$. (U)

19. Discuss the continuity of the cosine, cosecant, secant and cotangent functions. (U)

THREE MARK QUESTIONS

1. Discuss the continuity of the function f defined by $f(x)=\begin{cases} x+2, & if\ \ x\le 1 \\ x-2, & if\ \ x>1 \end{cases}$. (U)

2. Find all the points of discontinuity of the function f defined by $f(x)=\begin{cases} x+2, & if\ \ x<1 \\ 0, & if\ \ x=1 \\ x-2, & if\ \ x>1 \end{cases}$. (U)

3. Discuss the continuity of the function f defined by $f(x)=\begin{cases} x+2, & if\ \ x<0 \\ -x+2, & if\ \ x>0 \end{cases}$. (U)

4. Discuss the continuity of the function f defined by $f(x)=\begin{cases} x, & if\ \ x\ge 0 \\ x^2, & if\ \ x<0 \end{cases}$.(U)

5. Discuss the continuity of the sine function. (U)

6. Prove that the function defined by $f(x)=\tan x$ is a continuous function. (U)

7. Show that the function defined by $f(x)=\sin(x^2)$ is a continuous function. (U)

8. Is the function f defined by $f(x)=\begin{cases} x, & if\ \ x\le 1 \\ 5, & if\ \ x>0 \end{cases}$ continuous at $x=0$? At $x=1$? At $x=2$?

(U)

FOUR MARK QUESTIONS

1. Find all points of discontinuity of the greatest integer function defined by $f(x)=[x]$, where $[x]$ denotes the greatest integer less than or equal to x. (U)

2. Show that the function f defined by $f(x)=\left|1-x+|x|\right|$, where x is any real number, is a continuous function. (U)

3. Find all points of discontinuity of f, where f is defined by: $f(x)=\begin{cases} 2x+3, & if\ \ x\le 2 \\ 2x-3, & if\ \ x>2 \end{cases}$.

(U)

4. Find all points of discontinuity of f ,where f is defined by: $f(x)=\begin{cases} |x|+3, & if\ \ x\le -3 \\ -2x, & if\ \ -3<x<3 \\ 6x+2, & if\ \ x\ge 3 \end{cases}$

.(U)

5. Find all points of discontinuity of f, where f is defined by: $f(x)=\begin{cases} x+1, & if\ \ x\ge 1 \\ x^2+1, & if\ \ x<1 \end{cases}$.

(U)

6. Find all points of discontinuity of f ,where f is defined by: $f(x)=\begin{cases} \dfrac{x}{|x|}, & if\ \ x<0 \\ -1, & if\ \ x\ge 0 \end{cases}$. (U)

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7. Find all points of discontinuity of f, where f is defined by: $f(x)=\begin{cases}\dfrac{|x|}{x}, & if \quad x\neq 0\\ 0, & if \quad x=0\end{cases}$. (U)

8. Find all points of discontinuity of f, where f is defined by: $f(x)=\begin{cases}x^3-3, & if \quad x\le 2\\ x^2+1, & if \quad x>2\end{cases}$.

(U)

9. Find all points of discontinuity of f, where f is defined by: $f(x)=\begin{cases}x^{10}-1, & if \quad x\le 1\\ x^2, & if \quad x>1\end{cases}$.

(U)

10. Is the function defined by $f(x)=\begin{cases}x+5, & if \quad x\le 1\\ x-5, & if \quad x>1\end{cases}$ a continuous function? (U)

11. Discuss the continuity of the function f, where f is defined by:

$f(x)=\begin{cases}3, & if \quad 0\le x\le 1\\ 4, & if \quad 1<x<3\\ 5, & if \quad 3\le x\le 10\end{cases}$. (U)

12. Discuss the continuity of the function f, where f is defined by:

$f(x)=\begin{cases}2x, & if \quad x<0\\ 0, & if \quad 0\le x\le 1\\ 4x, & if \quad x>1\end{cases}$. (U)

13. Discuss the continuity of the function f, where f is defined by:

$f(x)=\begin{cases}-2, & if \quad x\le -1\\ 2x, & if \quad -1<x\le 1\\ 2, & if \quad x>1\end{cases}$. (U)

14. Find the relationship between 'a' and 'b' so that the function 'f' defined by

$f(x)=\begin{cases}ax+1, & if \quad x\le 3\\ bx+3, & if \quad x>3\end{cases}$ is continuous at $x=3$. (U)

15. For what value of λ is the function defined by $f(x)=\begin{cases}\lambda\left(x^2-2x\right), & \text{if} \quad x\le 0\\ 4x+1, & if \quad x>0\end{cases}$ is

continuous at $x=0$? What about continuity at $x=1$? (U)

16. Show that the function defined by $g(x)=x-[x]$ is discontinuous at all integral points. Here $[x]$ denotes the greatest integer less than or equal to x. (U)

17. Is the function defined by $f(x)=x^2-\sin x+5$ continuous at $x=\pi$? (U)

18. Find all the points of discontinuity of f, where $f(x)=\begin{cases}\dfrac{\sin x}{x}, & if \quad x<0\\ x+1, & if \quad x\ge 0\end{cases}$. (U)

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19. Determine if f defined by $f(x)=\begin{cases} x^2\sin\frac{1}{x}, & if \quad x\neq 0 \\ 0, & if \quad x=0 \end{cases}$ is continuous function? (U)

20. Examine the continuity of f, where f is defined by $f(x)=\begin{cases} \sin x-\cos x, & if \quad x\neq 0 \\ -1, & if \quad x=0 \end{cases}$.

(U)

21. Determine the value of k, if $f(x)=\begin{cases} \frac{k\cos x}{\pi-2x}, \; if \; x\neq\frac{\pi}{2} \\ 3, \quad if \quad x=\frac{\pi}{2} \end{cases}$ is continuous at $x=\frac{\pi}{2}$. (U)

22. Find the value of k if $f(x)=\begin{cases} kx^2, & if \quad x\leq 2 \\ 3, & if \quad x>2 \end{cases}$ is continuous at $x=2$. (U)

23. Find the value of k so that the function $f(x)=\begin{cases} kx+1, & if \quad x\leq\pi \\ \cos x, & if \quad x>\pi \end{cases}$, is continuous at

$x=\pi$. (U)

24. Find the value of k so that the function $f(x)=\begin{cases} kx+1, & if \quad x\leq 5 \\ 3x-5, & if \quad x>5 \end{cases}$, at $x=5$ is a

continuous function. (U)

25. Find the values of $a \; and \; b$ such that $f(x)=\begin{cases} 5, & if \quad x\leq 2 \\ ax+b, & if \; 2<x<10 \\ 21, & if \quad x\geq 10 \end{cases}$ is a continuous

functions. (U)

26. Show that the function defined by $f(x)=\cos\left(x^2\right)$ is a continuous function. (U)

27. Show that the function defined by $f(x)=|\cos x|$ is a continuous function. (U)

28. Examine that $\sin|x|$ is a continuous function. (U)

29. Find all the points of discontinuity of f defined by $f(x)=|x|-|x+1|$. (U)

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DIFFERENTIABILITY

ONE MARK QUESTIONS

1. Find the derivative of $y=\tan(2x+3)$. (U)

2. If $y=\sin(x^2+5)$, find $\frac{dy}{dx}$.(U)

3. If $y=\cos(\sin x)$, find $\frac{dy}{dx}$. (U)

4. If $y=\sin(ax+b)$, find $\frac{dy}{dx}$. (U)

5. If $y=\cos(\sqrt{x})$, find $\frac{dy}{dx}$. (U)

6. Find $\frac{dy}{dx}$, if $y=\cos(1-x)$.(U)

7. If $y=\log(\sin x)$, find $\frac{dy}{dx}$. (U)

8. Find $\frac{dy}{dx}$, *if* $x-y=\pi$. (U)

9. Find $\frac{dy}{dx}$, if $y=e^{-x}$. (U)

10. Find $\frac{dy}{dx}$, if $y=\sin(\log x),\ x>0$. (U)

11. Find $\frac{dy}{dx}$, if $y=\cos^{-1}(e^x)$.(K)

12. If $y=e^{\cos x}$, find $\frac{dy}{dx}$.(U)

13. Find $\frac{dy}{dx}$, if $y=e^{\sin^{-1}x}$. (A)

14. Find $\frac{dy}{dx}$, if $y=e^{x^3}$.(U)

15. Find $\frac{dy}{dx}$, if $y=\log(\log x),\ x>0$. (U)

16. Find $\frac{dy}{dx}$, if $y=x^3+\tan x$. (K)

17. Find $\frac{dy}{dx}$, if $y=x^2+3x+2$. (K)

18. Find $\frac{dy}{dx}$, if $y=x^{20}$. (K)

19. Find $\frac{dy}{dx}$, if $y=x\cos x$. (U)

20. Find $\frac{dy}{dx}$, if $y=\log x$. (U)

21. Find $\frac{dy}{dx}$, if $y=\tan^{-1}x$. (U)

22. Find $\frac{dy}{dx}$, if $y=\sin(\log x)$. (U)

23. If $y=e^{\log x}$, prove that $\frac{dy}{dx}=1$. (A)

24. Find $\frac{dy}{dx}$, if $y=5^x$. (U)

TWO MARK QUESTIONS

1. If $y=(2x+1)^3$, find $\frac{dy}{dx}$. (K)

2. Find the derivative of the function given by $f(x)=\sin(x^2)$. (U)

3. Find $\frac{dy}{dx}$, *if* $y+\sin y=\cos x$. (U)

4. Find $\frac{dy}{dx}$, if $2x+3y=\sin x$. (U)

5. Find $\frac{dy}{dx}$, if $2x+3y=\sin y$. (U)

6. Find $\frac{dy}{dx}$, if $ax+by^2=\cos y$. (U)

7. Find $\frac{dy}{dx}$, if $x^2+xy+y^2=100$. (U)

8. Find $\frac{dy}{dx}$, if $\sin^2 x+\cos^2 y=1$. (U)

9. If $\sqrt{x}+\sqrt{y}=10$, show that $\frac{dy}{dx}+\sqrt{\frac{y}{x}}=0$. (U)

10. Find $\frac{dy}{dx}$, if $y=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$. (U)

11. Find $\frac{dy}{dx}$, if $y=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$, $-\frac{1}{\sqrt{3}}<x<\frac{1}{\sqrt{3}}$. (U)

12. If $y=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $0<x<1$, find $\frac{dy}{dx}$. (U)

13. Find $\frac{dy}{dx}$, if $y=\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $0<x<1$. (U)

14. Find $\frac{dy}{dx}$, if $y=\cos^{-1}\left(\frac{2x}{1+x^2}\right)$, $-1<x<1$. (U)

15. Find $\frac{dy}{dx}$, if $y=\sin^{-1}\left(2x\sqrt{1-x^2}\right)$, $\frac{-1}{\sqrt{2}}<x<\frac{1}{\sqrt{2}}$. (U)

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16. Find $\frac{dy}{dx}$, if $y=\sec^{-1}\left(\frac{1}{2x^2-1}\right),\ 0<x<\frac{1}{\sqrt{2}}$. (U)

17. Find $\frac{dy}{dx}$, if $y=\log_a x$. (A)

18. Find $\frac{dy}{dx}$, if $y=\frac{e^x}{\sin x}$. (K)

19. Find $\frac{dy}{dx}$, if $y=\sin\left(\tan^{-1} e^{-x}\right)$. (A)

20. Find $\frac{dy}{dx}$, if $y=\log\left(\cos e^x\right)$. (A)

21. Find $\frac{dy}{dx}$, if $y=e^x+e^{x^2}+e^{x^3}.....+e^{x^5}$. (U)

22. Find $\frac{dy}{dx}$, if $y=\sqrt{e^{\sqrt{x}}},\ x>0$. (U)

23. Find $\frac{dy}{dx}$, if $y=\frac{\cos x}{\log x}, x>0$. (K)

24. Find $\frac{dy}{dx}$, if $y=\cos(\log x+e^x),\ x>0$. (U)

25. Differentiate a^x with respect to x, where a is a positive constant. (K)

26. Differentiate $x^{\sin x},\ x>0$ with respect to x. (U)

27. Differentiate $(\log x)^{\cos x}$ with respect to x. (U)

28. If $y=x^x$, find $\frac{dy}{dx}$. (U)

29. Differentiate $\left(x+\frac{1}{x}\right)^x$ w. r. to x. (U)

30. Find $\frac{dy}{dx}$, if $y=x^{\left(x+\frac{1}{x}\right)}$. (U)

31. Find $\frac{dy}{dx}$, if $y=(\log x)^x$ *OR* $y=x^{(\log x)}$. (U)

32. Find $\frac{dy}{dx}$, if $y=(\sin x)^x$ *OR* $y=\sin^{-1}\sqrt{x}$. (U)

33. Find $\frac{dy}{dx}$, if $y=x^{\sin x}$ *OR* $y=(\sin x)^{(\cos x)}$. (U)

34. Find $\frac{dy}{dx}$, if $y=log_7(log\ x)$. (A)

35. Find $\frac{dy}{dx}$, if $y=\cos^{-1}(\sin x)$. (U)

36. Find $\frac{dy}{dx}$, if $y=(3x^2-9x+5)^9$. (U)

37. Find $\frac{dy}{dx}$, if $y=\sin^3 x+\cos^6 x$. (U)

38. Find $\frac{dy}{dx}$, if $y=(5x)^{3\cos 2x}$. (U)

39. Find $\frac{dy}{dx}$, if $y=\sin^{-1}\left(x\sqrt{x}\right), 0\le x\le 1$.(K)

40. Find $\frac{dy}{dx}$, if $y=\frac{\cos^{-1}\frac{x}{2}}{\sqrt{2x+7}}$, $-2<x<2$. (K)

41. Find $\frac{dy}{dx}$, if $y=(\log x)^{\log x}$, $x>1$. (U)

42. Find $\frac{dy}{dx}$, if $y=\cos(a\cos x+b\sin x)$, for some constant 'a' and 'b'. (U)

43. Find $\frac{dy}{dx}$, if $y=x^3\log x$. (U)

44. Find $\frac{dy}{dx}$, if $y=e^x\sin 3x$.(U)

45. Find $\frac{dy}{dx}$, if $y=e^{6x}\cos 3x$. (U)

THREE MARK QUESTIONS

1. Differentiate $\sin(\cos(x^2))$ w. respect to x. (U)

2. If $y=\sec\left(\tan(\sqrt{x})\right)$, find $\frac{dy}{dx}$.(U)

3. If $y=\frac{\sin(ax+b)}{\cos(cx+d)}$, find $\frac{dy}{dx}$. (U)

4. If $y=\cos x^3\cdot\sin^2(x^5)$, find $\frac{dy}{dx}$. (U)

5. Prove that the function *f* given by $f(x)=|x-1|, x\in R$ is not differentiable at $x=1$. (K)

6. Prove that the greatest integer function defined by $f(x)=[x],\ 0<x<3$ is not differentiable $x=1$ *and* $x=2$. (A)

7. If $y=2\sqrt{\cot(x^2)}$, find $\frac{dy}{dx}$. (U)

8. Find $\frac{dy}{dx}$, *if* $x+\sin xy-y=0$. (K)

9. Find $\frac{dy}{dx}$, if $xy+y^2=\tan x+y$. (K)

10. Find $\frac{dy}{dx}$, if $x^3+x^2y+xy^2+y^3=81$. (K)

11. Find $\frac{dy}{dx}$, if $\sin^2 x + \cos xy = k$. (K)

12. Find the derivative of f given by $f(x) = \sin^{-1} x$ assuming it exists. (K)

13. Find the derivative of f given by $f(x) = \tan^{-1} x$ assuming it exists. (K)

14. Differentiate e^x w. r. to x from first principle method. (K)

15. Differentiate $\log_e x$ w. r. to x from first principle method. (K)

16. Differentiate $\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}$ with respect to x. (K)

17. Find $\frac{dy}{dx}$, *if* $y^x + x^y + x^x = a^b$. (K)

18. Find $\frac{dy}{dx}$, if $y = \cos x \cdot \cos 2x \cdot \cos 3x$. (K)

19. Differentiate $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$ with respect to x. (K)

20. Find $\frac{dy}{dx}$, if $x^x - 2^{\sin x}$. (K)

21. Find $\frac{dy}{dx}$, if $y = (x+3)^2 \cdot (x+4)^3 \cdot (x+5)^4$. (K)

22. Find $\frac{dy}{dx}$, if $x^y + y^x = 1$. (U)

23. Find $\frac{dy}{dx}$, if $x^y = y^x$. (U)

24. Find $\frac{dy}{dx}$, if $xy = e^{x-y}$. (U)

26. Find the derivative of the function given by $f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$ and hence find $f'(1)$. (K)

27. Differentiate $(x^2 - 5x + 8)(x^3 + 7x + 9)$ by using product rule. (K)

28. Find $\frac{dy}{dx}$, if $y = x^{x\cos x}$ *OR* $y = \frac{x^2+1}{x^2-1}$. (U)

29. Find $\frac{dy}{dx}$, if $y = (x\cos x)^x$ *OR* $y = (x\sin x)^{\frac{1}{x}}$. (U)

30. If u, v and w are functions of x, then show that

$$\frac{d}{dx}(uvw) = uv\frac{d}{dx}w + vw\frac{d}{dx}u + wu\frac{d}{dx}v$$

31. Find $\frac{dy}{dx}$, if $x = a\cos\theta,\ y = a\sin\theta$. (U)

32. Find $\frac{dy}{dx}$, if $x = at^2,\ y = 2at$. (U)

33. Find $\frac{dy}{dx}$, if $x=a\left(\theta+\sin\theta\right),\ y=a\left(1-\cos\theta\right)$. (U)

34. Find $\frac{dy}{dx}$, if $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$. (U)

35. If $x=a\,cos^3\theta\,and\ y=a\,sin^3\theta,$ prove that $\frac{dy}{dx}=-\sqrt[3]{\frac{y}{x}}$. (A)

36. Find $\frac{dy}{dx}$, if $x=2at^2,\ y=at^4$. (U)

37. Find $\frac{dy}{dx}$, if $x=a\cos\theta,\ y=b\cos\theta$. (U)

38. Find $\frac{dy}{dx}$, if $x=sin\,t,\ y=cos\,2t$.(U)

39. Find $\frac{dy}{dx}$, if $x=4t,\ y=\frac{4}{t}$. (U)

40. Find $\frac{dy}{dx}$, if $x=\cos\theta-\cos2\theta,\ y=\sin\theta-\sin2\theta$. (U)

41. If $x=a\left(\theta-\sin\theta\right)$ and $y=a\left(1+\cos\theta\right)$then prove that $\frac{dy}{dx}=-\cot\left(\frac{\theta}{2}\right)$.(A)

42. Find $\frac{dy}{dx}$, if $x=\frac{\sin^3 t}{\sqrt{\cos 2t}},\ y=\frac{\cos^3 t}{\sqrt{\cos 2t}}$. (U)

43. Find $\frac{dy}{dx}$, if $x=a\left(\cos t+\log\tan\frac{t}{2}\right),\ y=a\sin t$. (A)

44. Find $\frac{dy}{dx}$, if $x=a\,sec\,\theta,\ y=b\,tan\,\theta$.(U)

45. Find $\frac{dy}{dx}$, if $x=a\left(\cos\theta+\theta\sin\theta\right),\ y=a\left(\sin\theta-\theta\cos\theta\right)$. (U)

46. If $x=\sqrt{a^{\sin^{-1}t}}$ *and* $y=\sqrt{a^{\cos^{-1}t}}$, then prove that $\frac{dy}{dx}=-\frac{y}{x}$.(A)

48. If $x=a\left(\theta+\sin\theta\right)$ *and* $y=a\left(1-\cos\theta\right)$. Prove that $\frac{dy}{dx}=\tan\left(\frac{\theta}{2}\right)$.(A)

49. Verify Rolle's Theorem for the function $y=x^2+2,\ x\in[-2,2]$. (K) **"OR"**

Verify Rolle's Theorem for the function $y=x^2+2,\ a=-2\ and\ b=2$. (K)

50. Verify Mean Value Theorem for the function $y=x^2$ in the interval $[2,4]$.(K)

51. Verify Rolle's Theorem for the function $y=x^2+2x-8,\ x\in[-4,2]$. (K)

52. Verify Mean Value theorem, if $f(x)=x^2-4x-3$ in the interval $[a,b]$, where $a=1\ and\ b=4$. (K)

53. Verify Mean Value Theorem if $f(x)=x^3-5x^2-3x$ in the interval $[1,3]$. (K)

PUE

54. Find $\frac{dy}{dx}$, if $y=\sqrt{3x+2}+\frac{1}{\sqrt{2x^2+4}}$. (U)

55. Find $\frac{dy}{dx}$, if $y=e^{\sec^2 x}+3\cos^{-1}x$. (K)

56. Find $f'(x)$, if $f(x)=\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$.(A)

57. Find $f'(x)$, if $f(x)=\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$.(A)

58. Find $f'(x)$ if $f(x)=(\sin x)^{\sin x}$ for all $0<x<\pi$. (U)

59. For a positive constant 'a' find $\frac{dy}{dx}$, where $y=a^{t+\frac{1}{t}}$ and $y=\left(t+\frac{1}{t}\right)^a$. (U)

60. Differentiate $\sin^2 x$ with respect to $e^{\cos x}$. (K)

61. Find $\frac{dy}{dx}$, if $y=\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right], 0<x<\frac{\pi}{2}$.(A)

62. Find $\frac{dy}{dx}$, if $y=(\sin x-\cos x)^{(\sin x-\cos x)}$, $\frac{\pi}{4}<x<\frac{3\pi}{4}$. (K)

63. Find $\frac{dy}{dx}$, if $x^x+x^a+a^x+a^a$, for some fixed $a>0$ *and* $x>0$. (K)

64. Find $\frac{dy}{dx}$, if $y=(x)^{x^2-3}+(x-3)^{x^2}$, *for* $x>3$. (U)

65. Find $\frac{dy}{dx}$, if $y=12(1-\cos t),\ x=10(t-\sin t),\ -\frac{\pi}{2}<t<\frac{\pi}{2}$. (U)

66. Find $\frac{dy}{dx}$, if $y=\sin^{-1}x+\sin^{-1}\sqrt{1-x^2},\ 0<x<1$. (A)

67. If $x\sqrt{1+y}+y\sqrt{1+x}=0,$ *for* $-1<x<1$. Prove that $\frac{dy}{dx}=-\frac{1}{(1+x)^2}$.(K)

68. If $\cos y=x\cos(a+y),$ with $\cos a\neq\pm1$ prove that $\frac{dy}{dx}=\frac{\cos^2(a+y)}{\sin a}$.(K)

69. If $f(x)=|x|^3$, $f''(x)$ exists for all real x and find it. (U)

70. Using mathematical induction prove that $\frac{d}{dx}x^n=nx^{n-1}$ for all positive integer n.(U)

71. Using the fact that $\sin(A+B)=\sin A\cos B+\cos A\sin B$ and the differentiation, obtain the sum formula for cosines. (U)

72. Does there exists a function which is continuous everywhere but not differentiable at exactly two points? Justify your answer. (A)

73. If $y=\begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$, prove that, $\dfrac{dy}{dx}=\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}$. (S)

FIVE MARKS QUESTIONS

1. If $y=A\sin x+B\cos x$, then prove that $\dfrac{d^2y}{dx^2}+y=0$. (K)

2. If $y=3e^{2x}+2e^{3x}$, then prove that $\dfrac{d^2y}{dx^2}-5\dfrac{dy}{dx}+6y=0$. (K)

3. If $y=\sin^{-1}x$, then prove that $(1-x^2)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}=0$. (U)

4. If $y=5\cos x-3\sin x$, then prove that $\dfrac{d^2y}{dx^2}+y=0$. (U)

5. If $y=\cos^{-1}x$, *find* $\dfrac{d^2y}{dx^2}$ in terms if y alone. (S)

6. If $y=3\cos(\log x)+4\sin(\log x)$, show that $x^2 y_2+xy_1+y=0$. (U)

7. If $y=Ae^{mx}+Be^{nx}$, prove that $\dfrac{d^2y}{dx^2}-(m+n)\dfrac{dy}{dx}+(mn)y=0$. (K)

8. If $y=500e^{7x}+600e^{-7x}$, show that $y_2=49y$. (K)

9. If $e^y\,(x+1)=1$, show that $y_2=y_1^2$. (K)

10. If $e^y\left(x\ +\ 1\right)=1$, Prove that $\dfrac{dy}{dx}=-e^y$ hence prove that $\dfrac{d^2y}{dx^2}=\left(\dfrac{dy}{dx}\right)^2$. (K)

11. If $y=\left(\tan^{-1}x\right)^2$, show that $(1+x^2)^2y_2+2x(1+x^2)y_1=2$. (U)

12. If $y=e^{a\cos^{-1}x}$, $-1\le x\le 1$, show that $(1-x^2)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}-a^2y=0$. (A)

13. If $x=a(\cos t+t\sin t)$ and $y=a(\sin t-t\cos t)$, find $\dfrac{d^2y}{dx^2}$. (A)

14. If $\left(x-a\right)^2+\left(y-b\right)^2=c^2$, *for some* $c>0$, prove that $\dfrac{\left(1+{y_1}^2\right)^{\frac{3}{2}}}{y_2}$ is a constant independent of a and b. (A)

CHAPTER -06

APPLICATION OF DERIVATIVES

Rate of Change of Quantities:

FIVE MARK QUESTIONS

1. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.(U)

2. The radius of an air bubble is increasing at the rate of $\frac{1}{2} cm/s$. At what rate is the volume of the bubble increasing when the radius is $1cm$?(U)

3. A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$. Find the rate of change of its volume with respect to x.(U)

4. The length x of a rectangle is decreasing at the rate of $3\ cm/minute$ and the width y is increasing at the rate of $2\ cm/minute$. When $x=10\ cm\ and\ \ y=6\ cm$, find the rate of change of (i) the perimeter and (ii) the area of the rectangle.(U)

5.The length x of a rectangle is decreasing at the rate of $5\ cm/minute$ and the width y is increasing at the rate of $4\ cm/minute$. When $x=\ 8\ cm$ and $y=6\ cm$, find the rates of change of **(i)** the perimeter, **(ii)** the area of the rectangle.(U)

6. The volume of a cube is increasing at the rate of $8\ cm^3/s$. How fast is the surface area increasing when the length of an edge is $12\ cm$?(A)

7. The volume of a cube is increasing at a rate of 9 cubic centimeters persecond. How fast is the surface area increasing when the length of an edge is 10centimeters?(U)

8. An edge of a variable cube is increasing at the rate of $3\ cm/s$. How fast is the volume of the cube increasing when the edge is $10\ cm$ long?(U)

9. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic meter per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4m.(A)

10. A sand is pouring from a pipe at the rate of 12cm^3/s. The falling sand form a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base.

How fast is the height of the sand cone increasing when the height is $4cm$?(A)

11.A ladder $5m$ long is leaning against a wall. The bottom of the ladder is Pulled along the ground, Away from the wall at the rate of $2m/sec$.How fast is its height on the wall decreasing when the foot of the ladder is $4m$ away from the wall? (A)

12.A ladder 24 feet long leansagainst a vertical wall. Thelower end ismoving away at the rate of 3feet/sec. findthe rate at which the top of the ladder ismoving downwards, if its foot is 8 feet from the wall.(A)

13.A man of height 2 meters walks at a uniform speed of 5 km/hour, away from a lamp post which is 6 meters high. Find the rate at which the length of the his shadow increases.(A)

14.A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the $y-$coordinate is changing 8 times as fast as the $x-$coordinate.(U)

15. The total cost $C(x)$ in Rupees, associated with the production of x units of an item is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$. Find the marginal cost when 3 units are produced, where by marginal cost wemean the instantaneous rate of change of total cost at any level of output.(U)

16. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. Find the marginal revenue, when $x = 5$, where by marginal revenue we mean the rate of change of total revenue with respect to thenumber of items sold at an instant.(U)

17. The total cost $C(x)$ in Rupees associated with the production of x units of an item is given by $C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000$ .Find the marginal cost when 17 units are produced.(U)

18. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 13x^2 + 26x + 15$. Find the marginal revenue when $x = 7$.(U)

19. Find the total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. The marginal revenue, when $x = 15$.(U)

20. The radius of a circle is increasing uniformly at the rate of $3\ cm/s$. Find the rate at which the area of the circle is increasing when the radius is $10\ cm$.(U)

21. Find the rate of change of the area of a circle with respect to its radius r at $r = 6\ cm$.(U)

22. A stone is dropped into a quiet lake and waves move in circles at a speed of 4cm per second. At the instant, when the radius of the circular wave is $10\ cm$, how fast is the enclosed area increasing?(U)

PUE

23. A car starts from a point P at time $t = 0$ seconds and stops at point Q. The distance x , in meters, covered by it, in t seconds is given by $x = t^2\left(2 - \frac{t}{3}\right)$. Find the time taken by it to reach Q and also find distance between P and Q.(U)

24. Find the rate of change of the area of a circle per second with respect to its radius r when $r = 5\ cm$.(U)

25. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference?(U)

PUE

TANGENT AND NORMAL:

Two mark questions:

1. Find the slope of the tangent to the curve $y = x^3 - x$ at $x = 2$. (U)

2. Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at $x = 4$. (U)

3. Find the slope of the tangent to the curve $y = \dfrac{x-1}{x-2},\ x \neq 2$ at $x = 10$. (U)

4. Find the slope of the tangent to curve $y = x^3 - x + 1$ at the point whose $x-$coordinate is 2. (K)

5. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose $x-$coordinate is 3. (K)

6. Find the equation of the normal at the point (am^2, am^3) for the curve $ay^2 = x^3$.(U)

7. Find the slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$. (K)

THREE MARK QUESTIONS

1. Find the point at which the tangent to the curve $y = \sqrt{4x-3} - 1$ has its slope $\dfrac{2}{3}$. (K)

2. Find the equation of all lines having slope 2 and being tangent to the curve $y + \dfrac{2}{x-3} = 0$. (K)

3. Find points on the curve $\dfrac{x^2}{4} + \dfrac{y^2}{25} = 1$ at which the tangents are parallel to $x-axis$. (U)

4. Find points on the curve $\dfrac{x^2}{4} + \dfrac{y^2}{25} = 1$ at which the tangents are parallel to $y-axis$. (U)

5. Find the equation of the tangent to the curve $y = \dfrac{x-7}{(x-2)(x-3)}$ at the point where it cuts the $x-axis$. (U)

6. Find the equations of the tangent and normal to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2$ at $(1, 1)$. (U)

7. Find the equation of tangent to the curve given by $x = a\sin^3 t,\quad y = b\cos^3 t$ at a point where $t = \dfrac{\pi}{2}$. (K)

8. Find the slope of the normal to the curve $x = a\cos^3\theta, y = b\sin^3\theta$ at $\theta = \dfrac{\pi}{4}$. (K)

9. Find the slope of the normal to the curve $x = 1 - a\sin\theta,\ y = b\cos^2\theta$ at $\theta = \dfrac{\pi}{2}$. (K)

10. Find points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to the $X-axis$. (K)

11. Find a point on the curve $y = (x-2)^2$ at which the tangent is parallel to the chord joining the points $(2, 0)$ and $(4, 4)$. (S)

12. Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$. (K)

13. Find the equation of all lines having slope -1 that are tangents to the curve $y = \dfrac{1}{x-1},\ x \neq 1$. (K)

14. Find the equation of all lines having slope 2 which are tangents to the curve $y=\dfrac{1}{x-3}$, $x\neq 3$. (K)

15. Find the equations of all lines having slope 0 which are tangent to the curve $y=\dfrac{1}{x^2-2x+3}$. (K)

16. Find points on the curve $\dfrac{x^2}{9}+\dfrac{y^2}{16}=1$ at which the tangents are parallel to $X-axis$.(K)

17. Find points on the curve $\dfrac{x^2}{9}+\dfrac{y^2}{16}=1$ at which the tangents are parallel to $Y-axis$.(K)

19. Find the equations of the tangent and normal to the given curve $y=x^4-6x^3+13x^2-10x+5$ at $(0,5)$. (U)

20. Find the equations of the tangent and normal to the given curve $y=x^4-6x^3+13x^2-10x+5$ at $(1,3)$. (U)

21. Find the equations of the tangent and normal to the given curve $y=x^3$ at $(1,1)$. (U)

22. Find the equations of the tangent and normal to the given curve $y=x^3$ at $(0,0)$. (U)

23. Find the equations of the tangent and normal to the given curve $x=\cos t$, $y=\sin t$ at $t=\dfrac{\pi}{4}$. (K)

24. Find the equation of the tangent line to the curve $y=x^2-2x+7$ which is parallel to the line $2x-y+9=0$.(K)

25. Find the equation of the tangent line to the curve $y=x^2-2x+7$ which is perpendicular to the line $5y-15x=13$.(K)

26. Show that the tangents to the curve $y=7x^3+11$ at the points where $x=2$ and $x=-2$ are parallel. (K)

27. Find the points on the curve $y=x^3$ at which the slope of the tangent is equal to the $y-$coordinate of the point. (K)

28. For the curve $y=4x^3-2x^5$, find all the points at which the tangent passes through the origin. (K)

29. Find the points on the curve $x^2+y^2-2x-3=0$ at which the tangents are parallel to the $x-axis$. (K)

29. Find the equation of the normals to the curve $y=x^3+2x+6$ which are parallel to the line $x+14y+4=0$. (K)

30. Find the equations of the tangent and normal to the parabola $y^2=4ax$ at the point $(at^2,2at)$. (U)

31. Prove that the curves $x=y^2$ and $xy=k$ cut at right angle if $8k^2=1$. (A)

32. Find the equations of the tangent and normal to the hyperbola $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ at the point (x_0,y_0). (U)

33. Find the equation of the tangent to the curve $y=\sqrt{3x-2}$ which is parallel to the line $4x-2y+5=0$. (K)

34. Find the point at which the line $y=x+1$ is a tangent to the curve $y^2=4x$. (U)

35. Find the equation of the normal to curve $y^2=4x$ which passes through the point $(1,2)$. (U)

36. Show that the normal at any point θ to the curve $x=a\cos\theta+a\theta\sin\theta,\ y=a\sin\theta-a\theta\cos\theta$ is at a constant distance from the origin. (U)

37. Find the slope of the tangent to the curve $x=t^2+3t-8,\ y=2t^2-2t-5$ at the point $(2,-1)$. (K)

38. Find the value of 'm' so that the line $y=mx+1$ is a tangent to the curve $y^2=4x$. (U)

39. Find the equation of the normal to the curve $2y+x^2=3$ at the point $(1,1)$.(U)

40. Find the equation of the normal to the curve $x^2=4y$ at $(1,2)$. (U)

41. Find the points on the curve $9y^2=x^3$, where the normal to the curve makes equal intercepts with their axes. (U)

42. Find the equation of tangents to the curve $y=\cos(x+y),\ -2\pi\le x\le 2\pi$ that are parallel to the line $x+2y=0$. (K)

PUE

Increasing and Decreasing Functions

TWO MARK QUESTIONS

1.Show that the function given by $f(x)=7x-3$ is strictly increasing on R . (U)

2.Show that the function f given by $f(x)=x^3-3x^2+4x, x\in R$ is strictly increasing on R .(U)

3. Find the interval in which the function f given by $f(x)=2x^2-3x$ is strictly increasing. (U)

4. Prove that the function given by $f(x)=\cos x$ is strictly decreasing in $(0,\pi)$.(U)

5. Prove that the function given by $f(x)=\cos x$ is strictly increasing in $(\pi,\ 2\pi)$.(U)

6. Prove that the function given by $f(x)=\cos x$ is neither increasing nor decreasing in $(0,2\pi)$.(U)

7.Show that the function given by $f(x)=3x+17$ is strictly increasing on R .(U)

8.Show that the function given by $f(x)=e^{2x}$ is strictly increasing on R .(U)

9.Show that the function given by $f(x)=\sin x$ is strictly increasing in $\left(0,\frac{\pi}{2}\right)$.(U)

10.Show that the function given by $f(x)=\sin x$ is strictly decreasing in $\left(\frac{\pi}{2},\pi\right)$ (U)

11.Show that the function given by $f(x)=\sin x$ is neither increasing nor decreasing in $(0,\pi)$.(U)

12.Find the intervals in which the function f given by $f(x)=2x^2-3x$ is strictly increasing.(U)

13.Find the intervals in which the function f given by $f(x)=2x^2-3x$ is strictly decreasing.(U)

14.Prove that the logarithmic function is strictly increasing on $(0,\infty)$.(U)

15. Show that the function given by $f(x)=\cos x$ is strictly decreasing in $\left(0,\frac{\pi}{2}\right)$.(U)

16. Show that the function given by $f(x)=\cos 2x$ is strictly decreasing in $\left(0,\frac{\pi}{2}\right)$.(U)

17. Show that the function given by $f(x)=\cos 3x$ is strictly decreasing in $\left(0,\frac{\pi}{2}\right)$.(U)

18. Show that the function given by $f(x)=\tan x$ is strictly decreasing in $\left(0,\frac{\pi}{2}\right)$.(U)

19.Prove that the function f given by $f(x)=x^3-3x^2-3x-100$ is increasing in R .(U)

20.Prove that the function f given by $f(x)=x^2e^{-x}$ is increasing in $(0,2)$.(U)

THREE MARK QUESTIONS

1. Find the intervals in which the function f given by $f(x)=x^2-4x+6$ is strictly increasing. (K)

2. Find the intervals in which the function f given by $f(x)=x^2-4x+6$ is strictly decreasing. (K)

3. Find the intervals in which the function f given by $f(x)=4x^3-6x^2-72x+30$ is strictly increasing. (K)

4. Find the intervals in which the function f given by $f(x)=4x^3-6x^2-72x+30$ is strictly decreasing. (K)

5.Find intervals in which the function given by $f(x)=\sin 3x,\ x\in\left[0,\frac{\pi}{2}\right]$ is increasing. (U)

6.Find intervals in which the function given by $f(x)=\sin 3x,\ x\in\left[0,\frac{\pi}{2}\right]$ is decreasing. (U)

7.Find the intervals in which the function f given by $f(x)=2x^3-3x^2-36x+7$ is strictly increasing.(U)

8.Find the intervals in which the function f given by $f(x)=2x^3-3x^2-36x+7$ is strictly decreasing. (U)

9. Find the intervals in which the function f given by x^2+2x-5 is strictly increasing. (U)

10. Find the intervals in which the function f given by x^2+2x-5 is strictly decreasing. (U)

11. Find the intervals in which the function f given by $10-6x-2x^2$ is strictly increasing. (U)

12. Find the intervals in which the function f given by $10-6x-2x^2$ is strictly decreasing. (U)

13. Find the intervals in which the function f given by $6-9x-x^2$ is strictly increasing. (U)

14. Find the intervals in which the function f given by $6-9x-x^2$ is strictly decreasing. (U)

15. Prove that the function f given by $f(x)=x^2-x+1$ is neither strictly increasing nor strictly decreasing on $(-1,1)$. (U)

16. Show that the function given by $f(x)=x^{100}+\sin x-1$ is strictly decreasing in $(0,1)$. (K)

17. Show that the function given by $f(x)=x^{100}+\sin x-1$ is strictly decreasing in $\left(\frac{\pi}{2},\pi\right)$.(K)

18. Show that the function given by $f(x)=x^{100}+\sin x-1$ is strictly decreasing in $\left(0,\frac{\pi}{2}\right)$.(K)

19. Find the least value of 'a' such that the function f given by $f(x)=x^2+ax+1$ is strictly increasing on $(1,2)$. (K)

20. Prove that the function f given by $f(x)=\log(\sin x)$ is strictly increasing on $\left(0,\frac{\pi}{2}\right)$.(U)

21. Prove that the function f given by $f(x)=\log(\sin x)$ is strictly decreasing on $\left(\frac{\pi}{2},\pi\right)$. (U)

22. Prove that the function f given by $f(x)=\log(\cos x)$ is strictly decreasing on $\left(0,\frac{\pi}{2}\right)$.(U)

23. Prove that the function f given by $f(x)=\log(\cos x)$ is strictly increasing on $\left(\frac{\pi}{2},\pi\right)$. (U)

24. Find the intervals in which the function f given by $f(x)=\sin x+\cos x$, $0\le x\le 2\pi$ is strictly increasing or strictly decreasing.

25. Find the intervals in which the following functions are strictly increasing or decreasing:
(i) $-2x^3-9x^2-12x+1$ (ii) $(x+1)^3(x-3)^3$.

26. Show that $y=\log(1+x)-\frac{2x}{2+x}$, $x>-1$, is an increasing function of x throughout its domain.

27. Find the values of x for which $y=[x(x-2)]^2$ is an increasing function.

28. Prove that $y=\frac{4\sin\theta}{(2+\cos\theta)}-\theta$ is an increasing function of θ in $\left[0,\frac{\pi}{2}\right]$.

29. Find intervals in which the function given by $f(x)=\frac{3}{10}x^4-\frac{4}{5}x^3-3x^2+\frac{36}{5}x+11$ is

(a) strictly increasing **(b)** strictly decreasing.

30. Show that the function f given by $f(x)=\tan^{-1}(\sin x+\cos x)$, $x>0$ is always an strictly increasing function in $\left(0,\frac{\pi}{4}\right)$.

31. Find the intervals in which the function f given by $f(x)=\frac{4\sin x-2x-x\cos x}{2+\cos x}$ is

(i) increasing (ii) decreasing.

32. Find the intervals in which the function f given by $f(x)=x^3+\frac{1}{x^3}$, $x\neq 0$ is

(i) increasing (ii) decreasing.

ADDITIONAL QUESTIONS

1. Find the intervals in which the function f given by $f(x) = \sin x + \cos x$, $0 \le x \le 2\pi$ is strictly increasing or strictly decreasing.

2. Find the intervals in which the following functions are strictly increasing or decreasing: (i) $-2x^3 - 9x^2 - 12x + 1$ (ii) $(x+1)^3(x-3)^3$.

3. Show that $y = \log(1+x) - \dfrac{2x}{2+x}$, $x > -1$, is an increasing function of x throughout its domain.

4. Find the values of x for which $y = [x(x-2)]^2$ is an increasing function.

5. Prove that $y = \dfrac{4\sin\theta}{(2+\cos\theta)} - \theta$ is an increasing function of θ in $\left[0, \dfrac{\pi}{2}\right]$.

6. Find intervals in which the function given by

$f(x) = \dfrac{3}{10}x^4 - \dfrac{4}{5}x^3 - 3x^2 + \dfrac{36}{5}x + 11$ is **(a)** strictly increasing **(b)** strictly decreasing.

7. Show that the function f given by $f(x) = \tan^{-1}(\sin x + \cos x)$, $x > 0$ is always an strictly increasing function in $\left(0, \dfrac{\pi}{4}\right)$.

8. Find the intervals in which the function f given by

$f(x) = \dfrac{4\sin x - 2x - x\cos x}{2 + \cos x}$ is (i) increasing (ii) decreasing.

9. Find the intervals in which the function f given by $f(x) = x^3 + \dfrac{1}{x^3}$, $x \ne 0$ is (i) increasing (ii) decreasing.

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PUE

Maxima and Minima

TWO MARK QUESTIONS:

1.Find the maximum and the minimum values of the function f given by $f(x)=x^2,\ x\in R$. (U)

2.Find the maximum and minimum values of the function f given by $f(x)=|x|,\ x\in R$. (U)

3. Find the maximum and minimum values of the function f given by $f(x)=x,\ x\in(0,1)$. (K)

4.Prove that the function $f(x)=e^x$ do not have maxima or minima. (U)

5.Prove that the function $g(x)=\log x$ do not have maxima or minima. (U)

6.Prove that the function $h(x)=x^3+x^2+x+1$ do not have maxima or minima. (U)

7.It is given that at $x=1$, the function $f(x)=x^4-62x^2+ax+9$ attains its maximum value, on the interval $[0,2]$. Find the value of 'a'. (U)

8. Find all points of local maxima and local minima of the function f given by $f(x)=x^3-3x+3$. (U)

9. Find all the points of local maxima and local minima of the function $f(x)=2x^3-6x^2+6x+5$.(U)

10. Find local minimum value of the function f given by $f(x)=3+|x|,\ x\in R$.(U)

11. Find all the points of local maxima and local minima of the function $f(x)=2x^3-6x^2+6x+5$. (U)

12. Show that the function given by $f(x)=\dfrac{\log x}{x}$ has maximum at $x=e$.(U)

13. At what points in the interval $[0,2\pi]$, does the function $\sin 2x$ attain its maximum value? (U)

14. Prove that the function f(x)= e^x do not have maxima or minima.

15. Prove that the function f(x)=logx do not have maxima or minima.

THREE MARK QUESTIONS

1 . Find local maximum and local minimum values of the function f given by $f(x)=3x^4+4x^3-12x^2+12$ (U) 2. Find the maximum and minimum values of the function given by $f(x)=(2x-1)^2+3$. (U)

2. Find the maximum and minimum values of the function given by $f(x)=9x^2+12x+2$.(U)

3. Find the maximum and minimum values of the function given by $f(x)=-(x-1)^2+10$ (U)

4. Find the maximum and minimum values of the function given by $g(x)=x^3+1$.(U)

5. Find the maximum and minimum values of the function given by $f(x)=|x+2|-1$.(A)

6. Find the maximum and minimum values of the function given by $g(x)=-|x+1|+3$ (A)

7. Find the maximum and minimum values of the function given by $h(x)=\sin(2x)+5$.(U)

8. Find the maximum and minimum values of the function given by $h(x)=|\sin 4x+3|+5$.(U)

9. Find both the maximum value and the minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval $[0, 3]$.(U)

10. Find the maximum and minimum values of the function given by $h(x) = x + 1,\ x \in (-1, 1)$. (U)

11. Find the maximum value of $[x(x-1)+1]^{\frac{1}{3}},\ 0 \le x \le 1$. (A)

12. Find two positive numbers whose sum is 15 and the sum of whose squares is minimum. (A)

13. Find two numbers whose sum is 24 and whose product is as large as possible. (A)

14. Find two positive numbers x *and* y such that $x + y = 60$ and xy^3 is maximum. (A)

15. Find two positive numbers x *and* y such that their sum is 35 and the product x^2y^5 is a maximum. (A)

16. Find two positive numbers whose sum is 16 and the sum of whose cubes is minimum. (A)

17. Find the local maxima and local minima of the function $g(x) = x^3 - 3x$. Also find the local maximum and the local minimum values. (U)

18. Find the local maxima and local minima of the function $f(x) = x^2$. Also find the local maximum and the local minimum values. (U)

19. Find the local maxima and local minima of the function $h(x) = \sin x + \cos x,\ 0 < x < \frac{\pi}{2}$. Also find the local maximum and the local minimum values. (U)

20. Find the local maxima and local minima of the function $f(x) = x^3 - 6x^2 + 9x + 15$. Also find the local maximum and the local minimum values. . (U)

21. Find the local maxima and local minima of the function $g(x) = \frac{x}{2} + \frac{2}{x},\ x > 0$. Also find the local maximum and the local minimum values. (U)

22. Find the local maxima and local minima of the function $f(x) = \sin x - \cos x,\ 0 < x < 2\pi$. Also find the local maximum and the local minimum values. (U)

23. Find the local maxima and local minima of the function $f(x) = x\sqrt{1-x},\ x > 0$. Also find the local maximum and the local minimum values. (U)

24. Find the local maxima and local minima of the function $g(x) = \frac{1}{x^2+2}$. Also find the local maximum and the local minimum values. (U)

25. Find the absolute maximum value and the absolute minimum value of the function $f(x) = (x-1)^2 + 3,\ x \in [-3, 1]$. (U)

26. Find the absolute maximum value and the absolute minimum value of the function $f(x) = x^3,\ x \in [-2, 2]$. (U)

27. Find the absolute maximum and minimum values of a function $f(x) = 2x^3 - 15x^2 + 36x + 1$ on the interval $[1, 5]$. (U)

28. Find absolute maximum and minimum values of a function f given by

$f(x)=12x^{\frac{4}{3}}-6x^{\frac{1}{3}},\ x\in[-1,1]$. (U)

29. Find the absolute maximum value and the absolute minimum value of the function

$f(x)=\sin x+\cos x,\ x\in[0,\pi]$. (U)

30. Find the absolute maximum value and the absolute minimum value of the function

$f(x)=4x-\frac{1}{2}x^2,\ x\in\left[-2,\frac{9}{2}\right]$. (U)

31. Find the absolute maximum and minimum values of the function f given by

$f(x)=\cos^2 x+\sin x,\ x\in[0,\pi]$. (U)

32. Manufacturer can sell x items at a price of rupees $\left(5-\frac{x}{100}\right)$ each.

The cost price of x items is Rs $\left(\frac{x}{5}+500\right)$.

Find the number of items he should sell to earn maximum profit. (A)

33. Find the maximum profit that a company can make, if the profit function is given by

$p(x)=41-72x-18x^2$. (A)

34. What is the maximum value of the function $\sin x+\cos x$?(U)

35.Find the maximum value of $2x^3-24x+107$ in the interval $[1,3]$. (U)

36. Find the maximum value of $2x^3-24x+107$ in $[-3,-1]$.(U)

37.Find the maximum and minimum values of $x+\sin 2x$ on $[0,2\pi]$.(U)

38. Find the points at which the function f given by $f(x)=(x-2)^4(x+1)^3$ has (i) local maxima(ii) local minima. (U)

39.For all real values of x, find the minimum value of $\frac{1-x+x^2}{1+x+x^2}$. (U)

ADDITIONAL QUESTIONS

1. If length of three sides of a trapezium other than base are equal to 10cm, then find the area of the trapezium when it is maximum. (A)

2. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone. (A)

3. An Apache helicopter of enemy is flying along the curve given by $y=x^2+7$.

A soldier, placed at $(3,7)$, wants to shoot down the helicopter when it is nearest to him.

Find the nearest distance. (A)

4.A square piece of tin of side 18 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is the maximum possible. (A)

5.A rectangular sheet of tin $45\ cm$ by $24\ cm$ is to be made into a box without top, by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum ? (A)

6.Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.(A)

7.Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base. (A)

8.Of all the closed cylindrical cans (right circular), of a given volume of 100 cubic centimeters, find the dimensions of the can which has the minimum surface area? (A)

9.A wire of length $28\ m$ is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum? (A)

10.Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere. (A)

11.Show that the right circular cone of least curved surface area and given volume has an altitude equal to $\sqrt{2}$ time the radius of the base. (A)

12.Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.(A)

13.Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.(A)

14.An open topped box is to be constructed by removing equal squares from each corner of a 3 meter by 8 meter rectangular sheet of aluminum and folding up the sides. Find the volume of the largest such box. (A)

15.The two equal sides of an isosceles triangle with fixed base 'b' are decreasing at the rate of $3\ cm\ per\ second$. How fast is the area decreasing when the two equal sides are equal to the base ? (A)

16.Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with its vertex at one end of the major axis. (A)

17.A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is $2\ m$ and volume is $8\ m^3$. If building of tank costs $Rs\ 70\ per\ sq\ metres$ for the base and $Rs\ 45\ per\ square\ metre$ for sides. What is the cost of least expensive tank? (A)

18.The sum of the perimeter of a circle and square is k, where k is some constant .Prove that the sum of their areas is least when the side of square is double the radius of the circle. (A)

19.A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is $10\ m$. Find the dimensions of the window to admit maximum light through the whole opening. (A)

20. A point on the hypotenuse of a triangle is at distance ‘a’ and ‘b’ from the sides of the triangle .

Show that the maximum length of the hypotenuse is $\left(a^{\frac{2}{3}}+b^{\frac{2}{3}}\right)^{\frac{3}{2}}$.(A)

21.Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.(A)

22.Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume. (A)

23.Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle α is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.(A)

24. Find the point on the curve $x^2 = 2y$ which is nearest to the point $(0,5)$.(U)

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PUE

Approximations

TWO MARK QUESTIONS

1.Use differential to approximate $\sqrt{36.6}$. (U)

2.Using differentials, find the approximate value of $(25)^{\frac{1}{3}}$. (U)

3.Using differentials, find the approximate value of $\sqrt{25.3}$ up to 3 places of decimal.(K)

4.Use differential to approximate $\sqrt{25.3}$.(U)

5.Use differential to approximate $\sqrt{0.6}$.(U)

6. Using differentials, find the approximate value of $\sqrt{49.5}$. (U)

7. Using differentials, find the approximate value of $(0.009)^{\frac{1}{3}}$.(U)

8. Using differentials, find the approximate value of $(0.999)^{\frac{1}{10}}$.(U)

9.Use differential to approximate $(15)^{\frac{1}{4}}$.(U)

10.Use differential to approximate $(26)^{\frac{1}{3}}$.(U)

11.Use differential to approximate $(255)^{\frac{1}{4}}$.(U)

12.Use differential to approximate $(82)^{\frac{1}{4}}$ up to 3 places of decimal.(K)

13.Use differential to approximate $(401)^{\frac{1}{2}}$ up to 3 places of decimal.(K)

14.Use differential to approximate $(0.0037)^{\frac{1}{2}}$.(U)

15.Use differential to approximate $(26.57)^{\frac{1}{3}}$.(U)

16.Use differential to approximate $(81.5)^{\frac{1}{4}}$.(U)

17.Use differential to approximate $(3.968)^{\frac{3}{2}}$. (U)

18.Use differential to approximate $(32.15)^{\frac{1}{5}}$. (U)

19.Find the approximate change in the volume V of a cube of side x meters caused by increasing the side by 2%. (A)

PUE

20.If the radius of a sphere is measured as $9\ cm$ with an error of $0.03\ cm$, then find the approximate error in calculating its volume.(A)

21.Find the approximate change in the volume V of a cube of side x meters caused by increasing the side by 1%.(A)

22.Find the approximate change in the surface area of a cube of side x meters caused by decreasing the side by 1%.(A)

23.If the radius of a sphere is measured as $7\,m$ with an error of $0.02\ m$, then find the approximate error in calculating its volume.(A)

24.If the radius of a sphere is measured as $9\ m$ with an error of $0.03\ m$, then find the approximate error in calculating its surface area.(A)

25.Find the approximate change in the volume of a cube of side x meters caused by increasing the side by 3%. (A)

26.Find the approximate value of $f(3.02)$, where $f(x)=3x^2+5x+3$.(A)

27. Find the approximate value of $f(3.02)$, where $f(x)=3x^2+5x+3$.(A)

28.Find the approximate value of $f(2.01)$, where $f(x)=4x^2+5x+2$.(A)

29.Find the approximate value of $f(5.001)$, where $f(x)=x^3-7x^2+15$.(A)

30.Using differentials, find the approximate value of $\left(\frac{17}{81}\right)^{\frac{1}{4}}$.(U)

31.Using differentials, find the approximate value of $(33)^{-\frac{1}{5}}$.(U)

32.A circular disc of radius 3 cm is being heated. Due to expansion, its radius increases at the rate of $0.05\ cm/s$. Find the rate at which its area is increasing when radius is $3.2\ cm$.(A)

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CHAPTER -7

INTEGRALS

ONE MARKS QUESTIONS:

1. Find: $\int \sin mx dx$ (U)
2. Find: $\int (1-x)\sqrt{x} dx$ (U)
3. Find the anti-derivative of $\frac{1}{x\sqrt{x^2-1}}, x>1$ with respect to x (K)
4. Find: $\int e^x\left(\frac{x-1}{x^2}\right)dx$ (A)
5. Find: $\int \operatorname{cosec}x(\operatorname{cosec}x-\cot x)dx$ (U)
6. Find: $\int (2x-3\cos x+e^x)dx$. (U)
7. Evaluate : $\int \tan^2 2x \, dx$ (U)
8. Find: $\int \sin(2+5x) \, dx$ (U)
9. Find: $\int \frac{1-x}{\sqrt{x}}dx$ (A)
10. Find: $\int \sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right) dx$ (A)
11. Find: $\int e^x \sec x(1+\tan x)dx$ (A)
12. Find: $\int (\sin x+\cos x)dx$ (U)
13. Find the anti derivative of $x^2\left(1-\frac{1}{x^2}\right)$ with respect to x. (A)
14. Write the antiderivative of e^{2x} with respect to x. (A)
15. Find an anti-derivative of 3^x with respect to x. (A)
16. Find an anti-derivative of $\log_e x$ with respect to x. (A)
17. Find the anti-derivative of sin2x with respect to x (A)
18. Find the antiderivative of cos3x with respect to x. (A)
19. Find the antiderivative of $(ax+b)^2$ with respect to x. (A)
20. Find the anti derivative of $\sin 2x-4e^{3x}$ with respect to x. (A)

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21. Find the anti derivative of $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$ with respect to x. (A)

22. Find: $\int \frac{dx}{x^2-16}$ (U)

23. Find: $\int xe^x dx$ (U)

24. Find: $\int (4e^{3x}+1)dx$ (U)

25. Find: $\int (ax^2+bx+c)dx$ (U)

26. Find: $\int (2x^2+e^x)dx$ (U)

27. Find: $\int \frac{x^3+5x^2-4}{x^2}dx$ (U)

28. Find: $\int \frac{x^3+3x+4}{\sqrt{x}}dx$ (U)

29. Find: $\int \frac{x^3-x^2+x-1}{x-1}dx$ (U)

30. Find: $\int \log x dx$ (A)

31. Find: $\int \sqrt{ax+b}dx$ (A)

32. Find: $\int e^{2x+3}dx$ (A)

33. Find $\int \sqrt{x}(3x^2+2x+3)dx$ (U)

34. Find $\int (2x-3\cos x+e^x)dx$ (U)

35. Find $\int (2x^2-3\sin x+5\sqrt{x})dx$ (U)

36. Find $\int \sec x(\sec x+\tan x)dx$ (U)

37. Find $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x}dx$ (U)

38. Find: $\int \sin^{-1}(\cos x)dx$ (S)

39. Find $\int \sqrt{1+\cos 2x}dx$ (A)

40. Find $\int \sqrt{1-\cos 2x}dx$ (A)

41. Find $\int \sqrt{1+\sin 2x}dx$ (A)

PUE

42. Evaluate: $\int_{-\pi/2}^{\pi/2} \sin^7 x dx$ (S)

43. Evaluate: $\int_{0}^{2x} \cos^5 x dx$ (S)

44. Evaluate: $\int_{-\pi/2}^{\pi/2} \sin^3 x dx$ (S)

45. Evaluate: $\int_{a}^{b} x dx$ (U)

46. Evaluate: $\int_{0}^{5} (x+1) dx$ (U)

47. Evaluate: $\int_{2}^{3} x^2 dx$ (U)

48. Evaluate: $\int_{1}^{4} (x^2 - x) dx$ (U)

49. Evaluate: $\int_{-1}^{1} e^x dx$ (U)

50. Evaluate: $\int_{0}^{4} (x + e^{2x}) dx$ (U)

51. Evaluate: $\int_{0}^{4} \frac{dx}{16 + x^2}$ (U)

52. Evaluate : $\int_{1}^{\sqrt{3}} \frac{dx}{1 + x^2}$ (A)

53. Evaluate: $\int_{0}^{\pi} (\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2}) dx$ (S)

54. Evaluate: $\int_{2}^{3} \frac{dx}{x^2 - 1}$ (U)

55. Evaluate: $\int_{0}^{\pi/4} \sin 2x dx$ (A)

56. Evaluate: $\int_{0}^{\pi/2} \cos 2x dx$ (A)

57. Evaluate: $\int_{0}^{\pi/4} \tan x dx$ (A)

PUE

58. Evaluate: $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$ (U)

59. Evaluate: $\int (a^x e^x)\, dx$ (U)

60.Evaluaate: $\int \left(e^{2\log \sec x}\right) dx$ (U)

61.Evaluate; $\int (e^x - x^e + e^e)\ dx$ (U)

TWO MARKS QUESTIONS

1. Find $\int \frac{\sin^2 x}{1+\cos x} dx$ (A)

2. Find $\int \frac{1}{\sin^2 x \cos^2 x} dx$ (A)

3. Find $\int \frac{\cos 2x - \sin 2\alpha}{\cos x - \sin \alpha}\ dx$ (A)

4. Find $\int 2x \sin(x^2+1) dx$ (A)

5. Find $\int \frac{\sin(\tan^{-1} x)}{1+x^2} dx$ (A)

6. Find $\int \tan x dx$ (A)

7. Find $\int \cot x dx$ (A)

8. Find $\int \sec x dx$ (A)

9. Find $\int \cos ec x dx$ (A)

10. Find : $\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx$ (A)

11. Find: $\int \frac{dx}{x^2 - 6x + 13}$ (A)

12. Find: $\int x^2 e^{x^3} dx$ (A)

13. Find $\int \frac{2 - 3\sin x}{\cos^2 x} dx$ (A)

14. Find: $\int \frac{dx}{3x^2 + 13x - 10}$ (A)

15. Find $\int \frac{1 - \cos 2x}{1 + \cos 2x} dx$ (A)

16. Find $\int \frac{1-\cos 2x}{1+\cos 2x}dx$ (A)

17. Find $\int \frac{dx}{\sqrt{5x^2-2x}}$ (A)

18. Find $\int \tan^{-1}\left(\frac{\sin 2x}{1+\cos 2x}\right)dx$ (A)

19. Find $\int \tan^{-1}\left(\frac{\sin 2x}{1+\cos 2x}\right)dx$ (A)

20. Find: $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx$ (A)

21. Find $\int \frac{1-\sin x}{\cos^2 x}dx$ (A)

22. Find $\int \frac{3\cos x+4}{\sin^2 x}dx$ (A)

23. Find $\int \frac{1}{1-\cos x}\,dx$ (A)

24. Find $\int \frac{2x}{1+x^2}dx$ (A)

25. Find $\int \frac{(\log x)^2}{x}dx$ (A)

26. Find $\int \sin x \sin(\cos x)dx$ (A)

27. Find $\int \sin(ax+b)\cos(ax+b)dx$ (A)

28. Find $\int (4x+2)\sqrt{x^2+x+1}dx$ (A)

29. Find $\int \frac{x}{\sqrt{x+4}}dx$ (A)

30. Find $\int \frac{x^2}{(2+3x^3)^3}dx$ (A)

31. Find $\int \frac{dx}{x(\log x)^m}, x>0, m\neq 1$ (A)

32. Find $\int \frac{x}{9-4x^2}dx$ (A)

33. Find $\int \frac{x}{e^{x^2}}dx$ (A)

PUE

34. Find $\int x\sqrt{1+2x^2}\,dx$ (A)

35. Find $\int \frac{e^{\tan^{-1}x}}{1+x^2}dx$ (A)

36 . Find $\int \frac{e^{2x}-1}{e^{2x}+1}\,dx$ (A)

37.Find $\int \tan^2(2x-3)dx$ (A)

38. Find $\int \sec^2(7-4x)dx$ (A)

39. Find $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$ (A)

40. Find $\int \frac{1}{\cos^2 x(1-\tan x)^2}\,dx$ (A)

41. Find $\int \frac{2\cos x-3\sin x}{6\cos x+4\sin x}dx$ (A)

42. Find $\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx$ (A)

43. Find $\int \sqrt{\sin 2x}\cos 2x dx$ (A)

44. Find $\int \frac{\cos x}{\sqrt{1+\sin x}}dx$ (A)

45. Find $\int \cot x \log\sin x dx$ (A)

46. Find $\int \cos^2 x dx$ (A)

47. Find $\int \sin 2x\cos 3x dx$ (A)

48. Find $\int \sin^3 x dx$ (A)

49. Find $\int \log x dx$ (A)

50. Find $\int e^x(\sec x)(1+\tan x)dx$ (A)

51. Find $\int \frac{\cos 2x}{(\sin x+\cos x)^2}dx$ (A)

52. Find $\int \tan^{-1}x dx$ (A)

53. Find $\int \sin^3 x\cos^2 x dx$ (A)

PUE

54. Find $\int \frac{3x^2}{x^6+1}dx$ (A)

55. Find $\int \frac{1}{\sqrt{1+4x^2}}dx$ (A)

56. Find $\int \frac{1}{\sqrt{(2-x)^2+1}}dx$ (A)

57. Find $\int \frac{1}{\sqrt{9-25x^2}}dx$ (A)

58. Find $\int \frac{3x}{1+2x^4}dx$ (A)

59. Find $\int \frac{x^2}{1-x^6}\ dx$ (A)

60. Find $\int \frac{x-1}{\sqrt{x^2-1}}dx$ (A)

61. Find $\int \frac{x^2}{\sqrt{x^6+a^6}}dx$ (A)

62. Find $\int \frac{\sec^2 x}{\sqrt{\tan^2 x+4}}\ dx$ (A)

63. Find $\int \frac{dx}{\sqrt{x^2+2x+2}}$ (A)

64. Find $\int \frac{dx}{9x^2+6x+5}$ (A)

65. Find $\int \frac{dx}{\sqrt{7-6x-x^2}}$ (A)

66. Find $\int \frac{dx}{\sqrt{(x-1)(x-2)}}$ (A)

67. Find $\int \frac{dx}{\sqrt{8-3x-x^2}}$ (A)

68. Find $\int \frac{dx}{x^2+2x+2}$ (A)

69. Find $\int \frac{dx}{\sqrt{9x-4x^2}}$ (A)

70. Find $\int x\sin x dx$ (A)

PUE

71. Find $\int x\sin 3x dx$ (A)

72. Find $\int x^2 e^x dx$ (A)

73. Find $\int x\log x dx$ (A)

74. Find $\int x\log 2x dx$ (A)

75. Find $\int x^2 \log x dx$ (A)

76. Find $\int x\sec^2 x dx$ (A)

77. Find $\int \sqrt{x^2+2x+5} dx$ (A)

78. Find $\int \sqrt{3-2x-x^2} dx$. (A)

79. Find $\int \sqrt{4-x^2} dx$ (A)

80. Find $\int \sqrt{1-4x^2} dx$ (A)

81. Find $\int \sqrt{x^2+4x+6}\ dx$ (A)

82. Find $\int \sqrt{x^2+4x+1}\ dx$ (A)

83. Find $\int \sqrt{1-4x-x^2} dx$ (A)

84. Find $\int \sqrt{1+3x-x^2} dx$ (A)

85. Find $\int \sqrt{x^2+3x} dx$ (A)

86. Find $\int \sqrt{1+x^2} dx$ (A)

87. Find $\int \sqrt{7-8x+x^2} dx$ (A)

88. Find $\int \sqrt{1+\frac{x^2}{9}} dx$ (A)

89. Evaluate $\int_1^2 (4x^3-5x^2+6x+9) dx$ (A)

90 . Evaluate $\int_0^{\pi/4} \sin 2x dx$ (A)

91. Evaluate $\int_0^{\pi/2} \cos 2x dx$ (A)

PUE

92. Evaluate $\int_{0}^{\pi/2} \cos^2 x dx$ (A)

93. Evaluate $\int_{2}^{3} \frac{xdx}{x^2+1}$ (A)

94. Evaluate $\int_{0}^{1} xe^{x^2} dx$ (A)

95. Evaluate $\int_{0}^{2/3} \frac{dx}{4+9x^2}$ (A)

96. Evaluate $\int_{2}^{3} \frac{xdx}{x^2+1}$ (A)

97. Evaluate $\int_{-\pi/4}^{\pi/4} \sin^2 x dx$ (A)

98. Evaluate $\int_{-1}^{1} \sin^5 x \cos^4 x dx$ (A)

99. Evaluate $\int_{-\pi/2}^{\pi/2} \sin^7 x dx$ (A)

100. Evaluate $\int_{0}^{2\pi} \cos^5 x dx$ (A)

101. Evaluate $\int_{-\pi/2}^{\pi/2} (x^3 + x\cos x + \tan^5 x + 1) dx$ (A)

102. Find $\int \cos 6x \sqrt{1+\sin 6x} dx$ (A)

THREE MARKS QUESTION

1. *Find* the antiderivative of $f(x)=4x^3 - \frac{3}{x^4}$ such that $f(2)=0$ (A)

2. Find the antiderivative of F of f defined by $f(x)=4x^3-6$, where $F(0)=3$ (A)

2. *Find* $\int \frac{dx}{x+x\log x}$ (A)

3. Find $\int \frac{\tan^4 \sqrt{x} \sec^2 \sqrt{x}}{\sqrt{x}} dx$ (A)

4. Find $\int \frac{\sin x}{\sin(x+a)} dx$ (A)

5. Find $\int \frac{1}{1+\tan x} dx$ (A)

6. Find $\int \frac{e^{2x}-1}{e^{2x}+1}dx$ (A)

7. Find $\int \tan^2(2x-3)dx$ (A)

8. Find $\int \frac{1}{\cos^2 x(1-\tan x)^2}dx$ (A)

9. Find $\int \frac{1}{1-\tan x}dx$ (A)

10. Find $\int \frac{1}{!+\cot x}dx$ (A)

11.Find $\int \frac{(x+1)(x+\log x)^2}{x}dx$ (A)

12. Find $\int \frac{\sqrt{\tan x}}{\sin x \cos x}dx$ (A)

13. Find $\int \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8}dx$ (A)

14. Find $\int \frac{(1+\log x)^2}{x}dx$ (A)

15. Find $\int \sqrt{\sin 2x} \cos 2x dx$ (A)

16. Find $\int \frac{\sin x}{1+\cos x} dx$ (A)

17. Find $\int \frac{\sin x}{(1+\cos x)^2}dx$ (A)

18. Find $\int \frac{1}{1+\cot x}dx$ (A)

19. Find $\int \frac{(\sin^2 x-\cos^2 x)}{\sin^2 x \cos^2 x}dx$ (A)

20. Find $\int \frac{\sin^3 x+\cos^3 x}{\sin^2 x \cos^2 x}dx$ (A)

21. Find $\int x\sqrt{x+2}dx$ (A)

22. Find $\int \frac{x+2}{2x^2+6x+5} dx$ (A)

23. Find $\int \frac{x+3}{\sqrt{5-4x+x^2}} dx$ (A)

24. Find $\int \frac{dx}{\sqrt{(x-a)(x-b)}}$ (A)

25. Find $\int \frac{4x+1}{\sqrt{2x^2+x-3}}\,dx$ (A)

26. Find $\int \frac{x+2}{\sqrt{4x-x^2}}\,dx$ (A)

27. Find $\int \frac{x+2}{\sqrt{x^2+2x+3}}\,dx$ (A)

28. Find $\int \frac{x+3}{x^2-2x-5}dx$ (A)

29. Find $\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx$ (A)

30. Find $\int \frac{dx}{(x+1)(x+2)}$ (U)

31. Find $\int \frac{x^2+1}{x^2-5x+6}dx$ (A)

32. Find $\int \frac{3x-2}{(x+1)^2(x+3)}dx$ (A)

33. Find $\int \frac{x^2}{(x^2+1)(x^2+4)}dx$ (A)

34. Find $\int \frac{(3\sin\phi-2)\cos\phi}{5-\cos^2\phi-4\sin\phi}d\phi$ (S)

35. FInd $\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx$ (A)

36. Find $\int \frac{x}{(x+1)(x+2)}dx$ (A)

37. Find $\int \frac{dx}{x^2-9}$ (A)

38. Find $\int \frac{3x-1}{(x-1)(x-2)(x-3)}dx$ (A)

39. Find $\int \frac{3x-1}{(x-1)(x-2)(x-3)}dx$ (A)

40. Find $\int \frac{2x}{x^2+3x+2}\,dx$ (A)

PUE

41. Find $\int \frac{1-x^2}{x(1-2x)}dx$ (A)

42. Find $\int \frac{x}{(x^2+1)(x-1)}dx$ (A)

43. Find $\int \frac{3x+5}{x^3-x^2-x+1}dx$ (A)

44. Find $\int \frac{2x-3}{(x^2-1)(2x+3)}dx$ (A)

45. Find $\int \frac{5x}{(x+1)(x^2-4)}dx$ (A)

46. Find $\int \frac{x^3+x+1}{x^2-1}dx$ (A)

47. Find $\int \frac{2}{(1-x)(1+x^2)}dx$ (A)

48. Find $\int \frac{3x-1}{(x+2)^2}dx$ (A)

49. Find $\int \frac{dx}{x^4-1}$ (S)

50. Find $\int \frac{1}{x(x^n+1)}dx$ (S)

51. Find $\int \frac{\cos x}{(1-\sin x)(2-\sin x)}dx$ (S)

52. Find $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}dx$ (S)

53. Find $\int \frac{2x}{(x^2+1)(x^2+3)}dx$ (S)

54. Find $\int \frac{dx}{x(x^4-1)}$ (S)

55. Find $\int \frac{dx}{e^x-1}$ (A)

56. Find $\int \frac{xdx}{(x-1)(x-2)}$ (A)

57. Find $\int \frac{dx}{x(x^2+1)}$ (A)

PUE

58. Find $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$ (A)

59. Find $\int e^x \sin x dx$ (A)

60. Prove that $\int e^x[f(x)+f'(x)]dx = e^x + c$ (A)

61. Find $\int x\sin^{-1}x dx$ (A)

62. Find $\int x\tan^{-1}x dx$ (A)

63. Find $\int x\cos^{-1}x dx$ (A)

64. Find $\int x(\log x)^2 dx$ (A)

65. Find $\int (x^2+1)\log x dx$ (A)

66. Find $\int e^x(\sin x+\cos x)dx$ (A)

67. Find $\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$ (S)

68. Find $\int \frac{xe^x}{(1+x)^2}dx$ (S)

69. Find $\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$ (A)

70. Find $\int \frac{(x-3)e^x}{(x-1)^3}dx$ (S)

71. Find $\int e^{2x}\sin x dx$ (A)

72. Find $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$ (A)

73. Find $\int e^x \sec x(1+\tan x)dx$ (A)

74. Evaluate $\int_0^2 (x^2+1)dx$ as the limit of sum (S)

75. Evaluate $\int_a^b x dx$ as the limit of sum (S)

76. Evaluate $\int_0^5 (x+1)dx$ as the limit of sum (S)

PUE

77. Evaluate $\int_{2}^{3} x^2 dx$ as the limit of sum (S)

78. Evaluate $\int_{-1}^{1} e^x dx$ as the limit of sum (S)

79. Evaluate $\int_{1}^{4} (x^2 - x) dx$ as the limit of sum (S)

80. Evaluate $\int_{0}^{4} (x + e^{2x}) dx$ as the limit of sum (S)

81. Evaluate $\int_{4}^{9} \frac{\sqrt{x}}{(30 - x^{3/2})^2} dx$ (A)

82. Evaluate $\int_{1}^{2} \frac{x dx}{9(x+1)(x+2)}$ (A)

83. Evaluate $\int_{0}^{\pi/4} \sin^3 2t \cos 2t dt$ (A)

84. Evaluate $\int_{0}^{1} \frac{2x+3}{5x^2+1} dx$ (A)

85. Evaluate $\int_{1}^{2} \frac{5x^2}{x^2+4x+3} dx$ (A)

86. Evaluate $\int_{0}^{\pi/4} (2\sec^2 x + x^3 + 2) dx$ (A)

87. Evaluate $\int_{0}^{2} \frac{6x+3}{x^2+4} dx$ (A)

88. Evaluate $\int_{0}^{1} (xe^x + \sin\frac{\pi x}{4}) dx$ (A)

89. Evaluate $\int_{-1}^{1} 5x^4 \sqrt{x^5+1} dx$ (A)

90. Evaluate $\int_{0}^{1} \frac{\tan^{-1} x}{1+x^2} dx$ (A)

91. Evaluate $\int_{0}^{\pi/2} \sqrt{\sin\phi} \cos^5 \phi d\phi$ (A)

PUE

92. Evaluate $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$ (A)

93. Evaluate $\int_0^2 x\sqrt{x+2}\,dx$ (A)

94. Evaluate $\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}dx$ (A)

95. Evaluate $\int_0^2 \frac{dx}{x+4-x^2}$ (A)

96. Evaluate $\int_{-1}^1 \frac{dx}{x^2+2x+5}$ (A)

97. Evaluate $\int_0^2 \left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}dx$ (A)

98. Evaluate $\int_{1/3}^1 \frac{(x-x^3)^{1/3}}{x^4}dx$ (A)

99. If $\text{f(x)}=\int_0^x t\sin t\,dt$, then find the value of f '(x) (A)

100. Prove that $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx \text{ if f(2a-x) =f(x)} \\ 0 \text{ if f(2a-x) =-f(x)} \end{cases}$ (A)

101. Prove that $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$ (A)

102. Find $\int \frac{(x^4-x)^{1/4}}{x^5}dx$. (S)

103. Find $\int \frac{x^4 dx}{(x-1)(x^2+1)}$. (S)

104. Find $\int \left[\log(\log x)+\frac{1}{(\log x)^2}\right]dx$ (S)

105. Find $\int \left[\sqrt{\cot x}+\sqrt{\tan x}\right]dx$. (S)

106. Find $\int \frac{\sin 2x\cos 2x}{\sqrt{9-\cos^4(2x)}}dx$ (S)

PUE

107. Find $\int \frac{1}{x-x^3}dx$ (S)

108. Find $\int \frac{1}{\sqrt{x+a}+\sqrt{x+b}}dx$ (S)

109. Find $\int \frac{1}{x\sqrt{ax-x^2}}\,dx$ (S)

110. Find $\int \frac{1}{x^2(x^4+1)^{3/4}}\,dx$ (S)

111. Find $\int \frac{1}{x^{1/2}+x^{1/3}}dx$ (S)

112. Find $\int \frac{5x}{(x+1)(x^2+9)}\,dx$ (S)

113. Find $\int \frac{\sin x}{\sin(x-a)}\,dx$ (S)

114. Find $\int \frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}}dx$ (S)

115. Find $\int \frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}\,dx$ (S)

116. Find $\int \frac{1}{\cos(x+a)\cos(x+b)}dx$ (S)

117. Find $\int \frac{x^3}{\sqrt{1-x^8}}dx$ (S)

118. Find $\int \frac{e^x}{(1+e^x)(2+e^x)}dx$ (S)

119. Find $\int \frac{1}{(x^2+1)(x^2+4)}dx$ (S)

120. Find $\int \cos^3 x e^{\log\sin x}dx$ (S)

121. Find $\int e^{3\log x}(x^4+1)^{-1}\,dx$ (S)

122. Find $\int f'(ax+b)\left[f(ax+b)\right]^n\,dx$ (A)

123. Find $\int \frac{1}{\sqrt{\sin^3 x\sin(x+\alpha)}}dx$ (A)

124. Find $\int \frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}dx, x\in\left[0,1\right]$ (S)

125. Find $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}dx$ (S)

126. Find $\int \frac{2+\sin 2x}{1+\cos 2x}dx$ (S)

127. Find $\int \frac{x^2+x+1}{(x+1)^2(x+2)}dx$ (S)

128. Find $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}dx$ (S)

129. Find $\int \frac{\sqrt{x^2+1}\left[\log(x^2+1)-2\log x\right]}{x^4}dx$ (S)

130. Find $\int_{\pi/2}^{\pi} e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$ (S)

131. Find $\int_0^{\pi/4} \frac{\sin x\cos x}{\cos^4 x+\sin^4 x}dx$ (S)

132. Find $\int_0^{\pi/2} \frac{\cos^2 x}{\cos^2 x+4\sin^2 x}dx$ (S)

133. Find $\int_{\pi/6}^{\pi/3} \frac{\sin x+\cos x}{\sqrt{\sin 2x}}dx$ (S)

134. Find $\int_0^1 \frac{dx}{\sqrt{1+x}-\sqrt{x}}$ (S)

135. Find $\int_0^{\pi/4} \frac{\sin x+\cos x}{9+16\sin 2x}dx$ (S)

136. Find $\int_0^{\pi/2} \sin 2x\tan^{-1}(\sin x)dx$ (S)

137. Find $\int_1^4 \left[|x-1|+|x-2|+|x-3|\right]dx$ (S)

138. Prove that $\int_1^3 \frac{dx}{x^2(x+1)}=\frac{2}{3}+\log\frac{2}{3}$ (S)

139. Prove that $\int_0^1 xe^x dx=1$ (S)

140. Prove that $\int_0^{\pi/4} 2\tan^3 x dx=1-\log 2$ (S)

PUE

141. Prove that $\int_0^1 \sin^{-1} x dx = \frac{\pi}{2} - 1$ (S)

142. Evaluate $\int_0^1 e^{2-3x} dx$ as a limit of sum. (S)

143. Find $\int \frac{dx}{e^x + e^{-x}}$ (A)

144. Find $\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$ (A)

145. *If* f(a+b-x)=f(x), then find $\int_b^a x f(x) dx$ (A)

146. *Show* that $\int_0^a f(x) g(x) dx = 2\int_0^a f(x) dx$, if f and g are defined as f(x)=f(a-x) and g(x)+g(a-x)=4. (A)

147. Evaluate $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$ (S)

FIVE MARKS QUESTION

1. *Find* the integral of $\frac{1}{\sqrt{a^2 - x^2}}$ with respect to x and evaluate $\int \frac{dx}{\sqrt{2x - x^2}}$ (A)

2. *Find* the integral of $\frac{1}{\sqrt{a^2 - x^2}}$ with respect to x and evaluate $\int \frac{dx}{\sqrt{9 - 25x^2}}$ (A)

3. *Find* the integral of $\frac{1}{\sqrt{a^2 - x^2}}$ with respect to x and evaluate $\int \frac{dx}{\sqrt{8 + 3x - x^2}}$ (A)

4. *Find* the integral of $\frac{1}{\sqrt{a^2 - x^2}}$ with respect to x and evaluate $\int \frac{dx}{\sqrt{9x - 4x^2}}$ (A)

5. *Find* the integral of $\frac{1}{\sqrt{a^2 - x^2}}$ with respect to x and evaluate $\int \frac{dx}{\sqrt{7 - 6x - x^2}}$ (A)

6. *Find* the integral of $\frac{1}{x^2 - a^2}$ with respect to x and evaluate $\int \frac{dx}{x^2 - 16}$ (A)

7. *Find* the integral of $\frac{1}{x^2-a^2}$ with respect to x and evaluate $\int \frac{dx}{3x^2+13x-10}$ (A)

8. *Find* the integral of $\frac{1}{x^2-a^2}$ with respect to x and evaluate $\int \frac{dx}{x^2-16}$ (A)

9. *Find* the integral of $\frac{1}{a^2-x^2}$ with respect to x and evaluate $\int \frac{x^2dx}{1-x^6}$ (A)

10. *Find* the integral of $\frac{1}{a^2-x^2}$ with respect to x and evaluate $\int \frac{\sin xdx}{1-4\cos^2 x}$ (A)

11. *Find* the integral of $\frac{1}{x^2+a^2}$ with respect to x and evaluate $\int \frac{dx}{x^2-6x+13}$ (A)

12. *Find* the integral of $\frac{1}{x^2+a^2}$ with respect to x and evaluate $\int \frac{dx}{x^2+16}$ (A)

13. *Find* the integral of $\frac{1}{x^2+a^2}$ with respect to x and evaluate $\int \frac{dx}{9x^2+4}$ (A)

14. *Find* the integral of $\frac{1}{x^2+a^2}$ with respect to x and evaluate $\int \frac{dx}{2x^2+50}$ (A)

15. *Find* the integral of $\frac{1}{x^2+a^2}$ with respect to x and evaluate $\int \frac{3x^2dx}{x^6+1}$ (A)

16. *Find* the integral of $\frac{1}{x^2+a^2}$ with respect to x and evaluate $\int \frac{dx}{x^2+2x+2}$ (A)

17. *Find* the integral of $\frac{1}{x^2+a^2}$ with respect to x and evaluate $\int \frac{dx}{9x^2+6x+5}$ (A)

18. *Find* the integral of $\frac{1}{x^2+a^2}$ with respect to x and evaluate $\int \frac{3xdx}{1+2x^4}$ (A)

19. *Find* the integral of $\frac{1}{x^2+a^2}$ with respect to x and evaluate $\int \frac{\sin xdx}{1+\cos^2 x}$ (A)

20. *Find* the integral of $\frac{1}{x^2+a^2}$ with respect to x and evaluate $\int \frac{\cos xdx}{1+\sin^2 x}$ (A)

21. *Find* the integral of $\frac{1}{x^2+a^2}$ with respect to x and evaluate $\int \frac{dx}{a^2\cos^2 x+b^2\sin^2 x}$ (A)

22. *Find* the integral of $\frac{1}{\sqrt{x^2-a^2}}$ with respect to x and evaluate $\int \frac{dx}{\sqrt{(x-1)(x-2)}}$ (A)

23. *Find* the integral of $\frac{1}{\sqrt{x^2-a^2}}$ with respect to x and evaluate $\int \frac{dx}{\sqrt{(x-a)(x-b)}}$ (A)

PUE

24. *Find* the integral of $\frac{1}{\sqrt{x^2-a^2}}$ with respect to x and evaluate $\int \frac{dx}{\sqrt{5x^2-2x}}$ (A)

25. *Find* the integral of $\frac{1}{\sqrt{x^2+a^2}}$ with respect to x and evaluate $\int \frac{dx}{\sqrt{1+4x^2}}$ (A)

26. *Find* the integral of $\frac{1}{\sqrt{x^2-a^2}}$ with respect to x and evaluate $\int \frac{x^6 dx}{\sqrt{x^6+a^6}}$ (A)

27. *Find* the integral of $\frac{1}{\sqrt{x^2-a^2}}$ with respect to x and evaluate $\int \frac{\sec^2 x dx}{\sqrt{\tan^2 x+4}}$ (A)

28. *Find* the integral of $\frac{1}{\sqrt{x^2-a^2}}$ with respect to x and evaluate $\int \frac{dx}{\sqrt{(2-x)^2+1}}$ (A)

29. *Find* the integral of $\frac{1}{\sqrt{x^2-a^2}}$ with respect to x and evaluate $\int \frac{dx}{\sqrt{x^2+2x+2}}$ (A)

30. *Find* the integral of $\sqrt{x^2-a^2}$ with respect to x and evaluate $\int \sqrt{x^2+4x+1}dx$ (A)

31. *Find* the integral of $\sqrt{x^2-a^2}$ with respect to x and evaluate $\int \sqrt{x^2+3x}dx$ (A)

32. *Find* the integral of $\sqrt{x^2-a^2}$ with respect to x and evaluate $\int \sqrt{x^2-8x+7}dx$ (A)

33. *Find* the integral of $\sqrt{x^2+a^2}$ with respect to x and evaluate $\int \sqrt{x^2+4x+6}dx$ (A)

34. *Find* the integral of $\sqrt{x^2+a^2}$ with espect to x and evaluate $\int \sqrt{x^2+2x+5}dx$ (A)

35. *Find* the integral of $\sqrt{x^2+a^2}$ with espect to x and evaluate $\int \sqrt{1+\frac{x^2}{9}}dx$ (A)

36. *Find* the integral of $\sqrt{a^2-x^2}$ with espect to x and evaluate $\int \sqrt{4-x^2}dx$ (A)

37. *Find* the integral of $\sqrt{a^2-x^2}$ with espect to x and evaluate $\int \sqrt{3-2x-x^2}dx$ (A)

38. *Find* the integral of $\sqrt{a^2-x^2}$ with espect to x and evaluate $\int \sqrt{1-4x-x^2}dx$ (A)

39. *Find* the integral of $\sqrt{a^2-x^2}$ with espect to x and evaluate $\int \sqrt{1+3x-x^2}dx$ (A)

40. *Find* the integral of $\sqrt{a^2-x^2}$ with espect to x and evaluate $\int \sqrt{1-4x^2}dx$ (A)

41. Find the integral of $x\sqrt{1+x-x^2}$ with respect to x. (A)

42. Find the integral of $x\sqrt{x+x^2}$ with respect to x. (A)

43. Find the integral of $(x+1)\sqrt{2x^2+3}$ with respect to x. (A)

44. Find the integral of $(x+3)\sqrt{3-4x-x^2}$ with respect to x. (A)

SIX MARKS QUESTION

1. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\int_0^{\pi/4} \log(1+\tan x)dx$ (A)

2. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ (A)

3. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x+\cos^{3/2} x}dx$ (A)

4. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\int_0^{\pi/2} \frac{\cos^5 x}{\sin^5 x+\cos^5 x}dx$ (A)

5. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x+\cos^4 x}dx$ (A)

6. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\int_0^{a} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$ (A)

7. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\int_0^{\pi/2} \frac{\sin x-\cos x}{1+\sin x\cos x}dx$ (A)

8. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$ (A)

9. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\int_0^{\pi} \frac{x}{1+\sin x}dx$ (A)

10. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\int_0^{\pi/4} \log(1+\tan x)dx$ (A)

11. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$ (A)

12. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\int_0^{\pi/2} \log\sin x dx$ (A)

13. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\int_0^{\pi} \log(1+\cos x)dx$ (A)

PUE

14. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\int_0^{\pi/2} (2\log\sin x - \log\sin 2x)dx$ (A)

15. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}dx$ (A)

16. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\int_0^{\pi} \frac{x}{a^2\cos^2 x + b^2\sin^2 x}dx$ (A)

17. Prove that $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ and evaluate $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$. (A)

18. Prove that $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx \text{ if f(x) is even} \\ 0 \text{ if f(x) is odd} \end{cases}$ *hence* evaluate $\int_{-\pi/2}^{\pi/2} \sin^7 x dx$. (A)

19. Prove that $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx \text{ if f(x) is even} \\ 0 \text{ if f(x) is odd} \end{cases}$ hence evaluate $\int_{-\pi/4}^{\pi/4} \sin^2 x dx$ (A)

20.Prove that $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx \text{ if f(x) is even} \\ 0 \text{ if f(x) is odd} \end{cases}$ hence evaluate $\int_{-1}^{1} x^{17}\cos^4 x dx$. (A)

21. Prove that $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx \text{ if f(x) is even} \\ 0 \text{ if f(x) is odd} \end{cases}$ hence evaluate $\int_{-1}^{1} \sin^5 x\cos^4 x dx$. (A)

22. Prove that $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx \text{ if f(x) is even} \\ 0 \text{ if f(x) is odd} \end{cases}$ hence evaluate

$\int_{-\pi/2}^{\pi/2} (x^3 + x\cos x + \tan^5 x)dx$.

(A)

23. Prove that $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx \text{ if f(x) is even} \\ 0 \text{ if f(x) is odd} \end{cases}$ hence evaluate $\int_{-\pi/2}^{\pi/2} \sin^2 x dx$. (A)

24. Prove that $\int_b^a f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ hence evaluate $\int_{-1}^{2} |x^3 - x|dx$. (A)

25. Prove that $\int_b^a f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ hence evaluate $\int_{-5}^{5} |x+2|dx$. (A)

26. Prove that $\int_b^a f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ hence evaluate $\int_2^8 |x-5|dx$. (A)

27. Prove that $\int_b^a f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ hence evaluate $\int_0^4 |x-1|dx$ (A)

28. Prove that $\int_b^a f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ hence evaluate $\int_{-1}^{3/2} |x\sin(\pi x)|dx$ (S)

29. Prove that $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx \text{ if f(2a-x)=f(x)} \\ 0 \text{ if f(2a-x)=-f(x)} \end{cases}$ hence evaluate

$\int_0^\pi \frac{x}{a^2\cos^2 x + b^2\sin^2 x}dx$ (A)

30. Prove that $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx \text{ if f(2a-x)=f(x)} \\ 0 \text{ if f(2a-x)=-f(x)} \end{cases}$ hence evaluate $\int_0^\pi \frac{x\tan x}{\sec x + \tan x}dx$. (A)

31. Prove that $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx \text{ if f(2a-x)=f(x)} \\ 0 \text{ if f(2a-x)=-f(x)} \end{cases}$ hence evaluate $\int_0^\pi \log(1+\cos x)dx$. (A)

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CHAPTER -08

APPLICATION OF INTEGRALS

3 MARKS QUESTIONS

1. Find the area enclosed by the circle $x^2 + y^2 = a^2$. (A)

2. Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (A)

3. Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. (A)

4. Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$. (A)

5. Find the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the ordinates x=0 and x=ae, where $b^2 = a^2(1-e^2)$ and e<1. (A)

6. Find the area of the region bounded by the curve $y^2 = x$ and the lines x=1, x=4 and the x-axis in the first quadrant. (A)

7. Find the area of the region bounded by $y^2 = 9x$ and the lines x=2, x=4 and the x-axis in the first quadrant. (A)

8. Find the area of the region bounded by $x^2 = 4y$, y=2,y=4 and the y-axis in the first quadrant. (A)

9. Find the area of the region bounded by the curve $y=x^2$ and the line y=4. and the y-axis in the first quadrant. (A)

10. Find the area of the region bounded by the parabola $y=x^2$ and $y=|x|$. (S)

11. Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$. (A)

12. The area between $x = y^2$ and x=4 is divided into equal parts by the line x=a, find the value of a. (A)

13. Find the area of region bounded by the curve $y^2=4x$, y-axis and the line x=3. (A).

14. Find the area bounded by the curve $x^2=4y$ and the line x=4y-2. (A)

PUE

15. Find the area of the region bounded by the curve $y^2=4x$ and the line x=3. (A)

16. Find the area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$

and the lines x=0, and x =2.

17. Find the area of the region bounded by the curves $y = x^2 + 2, y = x, x = 0$ and x=3. (A)

18. Find the area of the region bounded by the curve $y^2= 4x$, y-axis and the line y= 3. (A)

19. Find the area of the parabola $y^2 = 4ax$ bounded by its latus rectum. (A)

20. Find the smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line x+y=2. (A)

21. Find the area lying between the curves $y^2 = 4x$ and y=2x . (A)

22. Find the area of the region enclosed by the parabola $x^2=y$, the line y=x+2 and the x-axis. (A)

23. Find the area under the given curves and given lines $y=x^2$,x=1,x=2 and x-axis. (A)

24. Find the area under the given curves and given lines $y=x^4$,x=1,x=5 and x-axis. (A)

25. Find the area between the curves y = x and $y = x^2$. (A)

26. Find the area of the region lying in the first quadrant and bounded by $y=4x^2$.x=0,y=1 and y=4. (A)

27. Find the area bounded by the curve y=cosx between x=0 and x=2 π . (A)

28.Find the area bounded by the curve y=sinx between x=0 and x=2 π . (A)

29. Find the area bounded by the y-axis, y= cosx and y=sinx when $0 \le x \le \frac{\pi}{2}$ (S)

30. Find the area bounded by the curves $\{(x, y) : y \ge x^2 \text{ and } y=|x|\}$. (S)

31. Find the area bounded by the curve $y=x^3$, the x-axis and the ordinates x=-2 and x=1. (A)

32. Find the area bounded by the curve y=x$|x|$, x axis and ordinates x=-1 and x=1. (A)

33.Sketch the graph of $y = |x+3|$ and evaluate $\int_{-6}^{0} |x+3| dx$. (S)

FIVE MARKS QUESTION

1. Find the area of the region in the first quadrant enclosed by the x-axis, the line y=x, and the circle $x^2 + y^2 = 32$. (A)

2. Find the area of the region in the first quadrant enclosed by x-axis, line x= $\sqrt{3}y$ and the circle $x^2 + y^2 = 4$. (A)

3. Find the area lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and inside of the parabola $y^2 = 4x$. (A)

PUE

4. Using integration find the area of the region bounded by the triangle whose vertices are (1, 0), (2,2) and (3,1). (A)

5. Find the area of the region enclosed between the two circles:

$x^2+y^2=4$ and $(x-2)^2+y^2=4$. (A)

6. Find the area of the circle $4x^2+4y^2=9$ which is interior to the parabola $x^2=4y$. (A)

7. Find the area bounded by curves $(x-1)^2+y^2=1$ and $x^2+y^2=1$. (A)

8. Using integration find the area of the region bounded by the triangle whose vertices are (-1, 0), (1,3) and (3,2). (A)

9. Using integration find the area of the triangular whose sides have the equations y=2x+1, y=3x+1 and x=4. (A)

10. Prove that the curves $y^2=4x$ and $x^2=4y$ divide the area of the square by x=0, x=4, y=4 and y=0 into three equal parts. (A)

11. Find the area of the region $\{(x,y): 0\le y\le x+1, 0\le x\le 2\}$. (A)

12. Find the area enclosed between the parabola $y^2=4ax$ and the line y=mx. (A)

13. Find the area enclosed by the parabola $4y=3x^2$ and the line 2y=3x+12. (A)

14. Find the area of the smaller region bounded by the ellipse

$\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$. (A)

15. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1$. (A)

16. Find the area of the region enclosed by the parabola $x^2=y$, the line y=x+2 and the x-axis. (A)

17. Using the method of integration find the area bounded by the curve $|x|+|y|=1$. (A)

18. Using the method of integration find the area of the triangle ABC, coordinates of whose vertices are A(2,0),B(4,5) and C(6,3). (A)

19. Using method of integration find the area of the region bounded by line 2x+y=4, 3x-2y=6 and x-3y+5=0 . (A)

20. Find the area of the region $\{(x,y): y^2\le 4x, 4x^2+4y^2\le 9\}$. (S)

21. Find the area of the circle $x^2+y^2=16$ exterior to the parabola $y^2=6x$. (A)

22 Find the area of the region bounded by the two parabolas $y=x^2$ and $y^2=x$. (A)

CHAPTER -09

DIFFERENTIAL EQUATIONS

TWO MARKS QUESTION

1. Find order and degree of the differential equation $\frac{dy}{dx}-\cos x=0$ (U)

2. Find order and degree of the differential equation $xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$ (U)

3. Find order and degree of the differential equation $y'''+y^2+e^{y'}=o$ (U)

4. Find order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2+\sin\left(\frac{d^3y}{dx^3}\right)=0$ (U)

5. Find order and degree of the differential equation $\left(\frac{ds}{dt}\right)^4+3s\frac{d^2s}{dt^2}=0$ (U)

6. Find order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2+\cos\left(\frac{dy}{dx}\right)=0$ (U)

7. Find order and degree of the differential equation $y'+5y=0$ (U)

8. Find order and degree of the differential equation $\frac{d^2y}{dx^2}=\cos 3x+\sin 3x$ (U)

9. Find order and degree of the differential equation $(y''')^2+(y'')^3+(y')^4+y^5=0$ (U)

10. Find order and degree of the differential equation $y'+y=e^x$ (U)

11. Find order and degree of the differential equation $y'''+(y')^2+2y=0$ (U)

12. Find order and degree of the differential equation $y''+2y'+\sin y=0$ (U)

13. Find order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)^2+\sin\left(\frac{dy}{dx}\right)+1=0$ (U)

14. Find order and degree of the differential equation $2x^2\left(\frac{d^2y}{dx^2}\right)-3\left(\frac{dy}{dx}\right)+y=0$ (U)

15. Find order and degree of the differential equation $\left(\frac{dy}{dx}\right)^3 - 4\left(\frac{dy}{dx}\right)^2 + 7y = \sin x$ (U)

16. Find order and degree of the differential equation $\frac{d^4y}{dx^4} - \sin\left(\frac{d^3y}{dx^3}\right) = 0$ (U)

17. Find order and degree of the differential equation $\frac{d^2y}{dx^2} + 5x\left(\frac{dy}{dx}\right)^2 - 6y = \log x$ (U)

18. Find the number of arbitrary constants in the general solution of differential equation of fourth order also find the number of arbitrary constants in the particular solution of differential equation of third order. (U)

19. Find the general solution of a differential equation: $\frac{dx}{dy} + P_1x = Q_1$. (U)

THREE MARKS QUESTIONS

1. Verify that the function y= e^{-3x} is a solution of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$ (A)

2. Verify that the function y= a cosx +b sinx, where a,b $\in$ R is a solution of the differential equations $\frac{d^2y}{dx^2} + y = 0$. (A)

3. Verify that the function y= $e^x + 1$ is a solution of the differential equation $y'' - y' = 0$. (A)

4. Verify that the function $y = x^2 + 2x + c$ is a solution of the differential equation y'-2x-2=0. (A)

5. Verify that the function y=cosx+C is a solution of the differential equation y'+sinx=0. (A)

6. Verify that the function $y = \sqrt{1+x^2}$ is a solution of the differential equation $y' = \frac{xy}{1+x^2}$. (A)

7. Verify that the function y=Ax is a solution of the differential equation xy'=y(x $\neq 0$). (A)

8. Verify that the function $y = x\sin x$ is a solution of the differential equation $xy' = y + x\sqrt{x^2 - y^2}$ $(x \neq 0$ and $x>y$ or $x<-y)$ (A)

9. Verify that the function xy=logy+C is a solution of the differential equation $y' = \frac{y^2}{1-xy}$ $(xy \neq 1)$ (A)

10. Verify that the function y- cosy =x is a solution of the differential equation
(ysiny+cosy+x)y'=y. (A)

11. Verify that the function $x+y=\tan^{-1}y$ is a solution of the differential equation
$y^2y' + y^2 + 1 = 0$. (A)

12. Verify that the function $y=\sqrt{a^2-x^2}\ x \in (-a,a)$ is a solution of the differential equation
$x+y\frac{dy}{dx} = 0(y \neq 0)$. (A)

13. Verify that the function $y=ae^x+be^{-x}+x^2$ is a solution of the differential equation
$x\frac{d^2y}{dx^2}+2\frac{dy}{dx}-xy+x^2-2=0$ (A)

14. Verify that the function $y=e^x(a\cos x+b\sin x)$ is a solution of the differential equation .
$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$. (A)

15. Verify that the function y=xsin3x is a solution of the differential equation
$\frac{d^2y}{dx^2}+9y-6\cos 3x=0$. (A)

16. Verify that the function $x^2=2y^2\log y$ is a solution of the differential equation
$(x^2+y^2)\frac{dy}{dx}-xy=0$. (A)

17. Form the differential equation representing the family of curves y=asin(x+b), where a,b are arbitrary constants.
(A)

18. Form the differential equation representing the family of ellipses having foci on x-axis and centre at the origin. (A)

19. Form the differential equation of the family of circles touching the x-axis at origin. (A)

20. Form the differential equation representing the family of parabolas having vertex at origin and axis along positive direction of x-axis. (A)

21. Form the differential equation representing the family of circles touching the y-axis at origin. (A)

22. Form the differential equation representing the family of parabolas having vertex at origin and x-axis along positive y-axis. (A)

23. Form the differential equation representing the family of ellipses having foci on y-axis and centre at origin. (A)

24. Form the differential equation representing the family of hyperbolas having foci on x-axis and centre at origin. (A)

25. Form the differential equation representing the family of circles having centre on y-axis and radius 3 units. (A)

26. Form the differential equation representing the family of curves given by $(x-a)^2 + 2y^2 = a^2$, where a is an arbitrary constant. (A)

27. Form the differential equation of the family of circles in the first quadrant which touch the coordinate axes. (A)

FIVE MARKS QUESTIONS

1. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{x+1}{2-y}, (y \neq 2)$ (A)

2. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$. (A)

3. Find the particular solution of the differential equation

   $\frac{dy}{dx} = -4xy^2$ given that y=1, when x=0. (A)

4. Find the equation of the curve passing through the point (1, 1) whose differential equation is $xdy = (2x^2+1)dx (x \neq 0)$. (A)

5. Find the equation of a curve passing through the point (-2, 3), given that the slope of the tangent to the curve at any point (x, y) is $\frac{2x}{y^2}$. (A)

6. In a bank, principal increases continuously at the rate of 5% per year. In how many years Rs.1000 double itself? (A)

7. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$. (A)

8. Find the general solution of the differential equation $\frac{dy}{dx} = \sqrt{4-y^2} (-2 < y < 2)$. (A)

9. Find the general solution of the differential equation $\frac{dy}{dx} + y = 1 (y \neq 1)$. (A)

10. Find the general solution of the differential equation $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$. (A)

11. Find the general solution of the differential equation $(e^x + e^{-x})dy - (e^x - e^{-x})dx = 0$ (A)

12. Find the general solution of the differential equation $\frac{dy}{dx}=(1+x^2)(1+y^2)$. (A)

13. Find the general solution of the differential equation $y\log y dx - x dy = 0$. (A)

14. Find the general solution of the differential equation $x^5\frac{dy}{dx}=-y^5$. (A)

15. Find the general solution of the differential equation $\frac{dy}{dx}=\sin^{-1}x$. (A)

16. Find the general solution of the differential equation

$\mathrm{e}^x \tan y dx+(1-e^x)\sec^2 y dy=0$. (A)

17. Find the general solution of the differential equation $\frac{dy}{dx}+\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}=0$. (A)

18. Find the general solution of the differential equation $\frac{dy}{dx}=e^{x+y}$. (A)

19. Find the particular solution of the differential equation

$(x^3+x^2+x+1)\frac{dy}{dx}=2x^2+x; y=1 \text{ when } x=0$. (A)

20. Find the particular solution of the differential equation $\mathrm{x}(\mathrm{x}^2-1)\frac{dy}{dx}=1; y=0 \text{ when } x=2$. (A)

21. Find the particular solution of the differential equation

$\cos\left(\frac{dy}{dx}\right)=a(a\in R); y=2 \text{ when } x=0$. (A)

22. Find the particular solution of the differential equation $\frac{dy}{dx}=y\tan x; y=1 \text{ when } x=0$. (A)

23. Find the particular solution of the differential equation

$(1+e^{2x})dy+(1+y^2)e^x dx,$ given that y=1 when x=0 (A)

24. Find the particular solution of the differential equation $(x-y)(dx+dy)=dx-dy$ given that y=-1, when x=0. (A)

25. Find the equation of a curve passing through the point(0,0) and whose differential equation is $y'=e^x\sin x$. (A)

26. For the differential equation $xy\frac{dy}{dx}=(x+2)(y+2)$, find the solution curve passing through the point (1,-1). (A)

27. Find the equation of curve passing through the point (0,-2) given that at any point (x,y) on the curve, the product of the slope of its tangent and y coordinate of the point is equal to the x coordinate of the point. (A)

28. At any point (x,y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contanct to the point (-4,-3). Find the equation of the curve given that it passes through (-2,1). (A)

29. The volume of spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of balloon after t seconds. (A)

30. In a bank, principal increases continuously at the rate of r% per year. Find the value of r if Rs.100 if Rs.100 double itself in 10 years($\log_e 2=0.6931$). (A)

31. In a bank principal increases continuously at the rate of 5% per year. An amount of Rs.1000 is deposited with this bank, how much will it worth after 10 years ($e^{0.5}=1.648$). (A)

32. In a culture, the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number present? (A)

33. Show that the differential equation $(x-y)\frac{dy}{dx}=x+2y$ is homogeneous and solve it. (S)

34. Show that the differential equation $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx}=y\cos\left(\frac{y}{x}\right)+x$ is homogeneous and solve it.

(S)

35. Show that the differential equation $2ye^{\frac{x}{y}}dx+(y-2xe^{\frac{x}{y}})dy=0$ is homogeneous and find its particular solution, given that, x=0 when y=1. (S)

36. Show that the family of curves for which the slope of the tangent at any point (x,y) on it is $\frac{x^2+y^2}{2xy}$, is given by $x^2-y^2=cx$

37. Show that the differential equation $(x^2+xy)dy=(x^2+y^2)dx$ is homogeneous and solve it. (S)

38. Show that the differential equation $y'=\frac{x+y}{x}$ is homogeneous and solve it. (S)

39. Show that the differential equation $(x-y)dy-(x+y)dx=0$ is homogeneous and solve it. (S)

40. Show that the differential equation $(x^2 - y^2)dx + 2xydy = 0$ is homogeneous and solve it. (S)

41. Show that the differential equation $x^2 \frac{dy}{dx} = x^2 - 2y^2 + xy$ is homogeneous and solve it. (S)

42. Show that the differential equation $xdy - ydx = \sqrt{x^2 + y^2}dx$ is homogeneous and solve it. (S)

43. Show that the differential equation

$\left\{x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right\}ydx = \left\{y\sin\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)\right\}xdy$ is homogeneous and solve it.

(S)

44. Show that the differential equation $x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0$ is homogeneous and solve it. (S)

45. Show that the differential equation $ydx + x\log\left(\frac{y}{x}\right)dy - 2xdy = 0$

is homogeneous and solve it. (S)

46. Show that the differential equation $(1 + e^{\frac{x}{y}})dx + e^{\frac{x}{y}}(1 - \frac{x}{y})dy = 0$

is homogeneous and solve it. (S)

47. For, (x+y)dy+(x-y)dx=0, find the particular solution satisfying the given condition, y=1 when x=1. (S)

48. For $x^2dy + (xy + y^2)dx$, find the particular solution satisfying the given condition, y=1 when x=1. (S)

49. For, $\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + xdy = 0;$ find the particular solution satisfying the given condition,

$y = \pi/4$ when x=1. (S)

50. For, $\frac{dy}{dx} - \frac{y}{x} + \cos ec\left(\frac{y}{x}\right) = 0;$ find the particular solution satisfying the given condition, y=0 when x=1. (S)

51. For, $2xy + y^2 - 2x^2\frac{dy}{dx} = 0;$ find the particular solution satisfying the given condition,

y=2 when x=1. (S)

52. Find the general solution of the differential equation $\frac{dy}{dx} - y = \cos x$. (A)

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53. Find the general solution of the differential equation $x\frac{dy}{dx}+2y=x^2(x\neq 0)$. (A)

54. Find the general solution of the differential equation $ydx-(x+2y^2)dy=0$. (A)

55. Find the general solution of the differential equation $\frac{dy}{dx}+2y=\sin x$. (A)

56. Find the general solution of the differential equation $\frac{dy}{dx}+3y=e^{-2x}$. (A)

57. Find the general solution of the differential equation $\frac{dy}{dx}+\frac{y}{x}=x^2$. (A)

58. Find the general solution of the differential equation $\frac{dy}{dx}+(\sec x)y=\tan x(0\leq x\leq \pi/2)$. (A)

59. Find the general solution of the differential equation $\cos^2 x\frac{dy}{dx}+y=\tan x(0\leq x\leq \pi/2)$. (A)

60. Find the general solution of the differential equation $x\frac{dy}{dx}+2y=x^2\log x$. (A)

61. Find the general solution of the differential equation $x\log x\frac{dy}{dx}+y=\frac{2}{x}\log x$. (A)

62. Find the general solution of the differential equation $(1+x^2)dy+2xydx=\cot xdx(x\neq 0)$. (A)

63. Find the general solution of the differential equation $x\frac{dy}{dx}+y-x+xy\cot x=0(x\neq 0)$. (A)

64. Find the general solution of the differential equation $(x+y)\frac{dy}{dx}=1$. (A)

65. Find the general solution of the differential equation $ydx+(x-y^2)dy=0$. (A)

66. Find the general solution of the differential equation $(x+3y^2)\frac{dy}{dx}=y(y>0)$. (A)

67. For, $\frac{dy}{dx}+2y\tan x=\sin x$, find the particular solution satisfying the given condition y=0 when x=$\frac{\pi}{3}$. (A)

68. For, $(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{1+x^2}$, find the particular solution satisfying the given condition y=0 when x=1. (A)

69. For, $\frac{dy}{dx}-3y\cot x=\sin 2x$ find the particular solution satisfying the given condition y=2 when x=$\frac{\pi}{2}$. (A)

70. Find the equation of a curve passing through the origin given that the slope of the tangent to the curve at any point (x,y) is equal to the sum of the coordinates of the point. (S)

71. Find the equation of a curve passing through the point (0,2) given that the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5. (S)

72. Find the general solution of the differential equation $x\frac{dy}{dx}-y=2x^2$. (A)

73. Find the general solution of the differential equation $(1-y^2)\frac{dx}{dy}+yx=ay(-1<y<1)$. (A)

74. Verify that the function $y=c_1e^{ax}\cos bx+c_2e^{ax}\sin bx,$ where c_1, c_2 are arbitrary constants is a solution of the differential equation: $\frac{d^2y}{dx^2}-2a\frac{dy}{dx}+(a^2+b^2)y=0$. (S)

75. Form the differential equation of the family of circles in the second quadrant and touching the coordinate axes. (S)

76. Find the particular solution of the differential equation: $\log\left(\frac{dy}{dx}\right)=3x+4y$ given that y=0 when x=0. (S)

77. Solve the differential equation: $(xdy-ydx)y\sin\left(\frac{y}{x}\right)=(ydx+xdy)x\cos\left(\frac{y}{x}\right)$. (S)

78. Solve the differential equation: $(\tan^{-1}y-x)dy=(1+y^2)dx$. (S)

79. Prove that $x^2-y^2=c(x^2+y^2)^2$ is the general solution of differential equation $(x^3-3xy^2)dx=(y^3-3x^2y)dy$, where c is parameter. (S)

80. Show that the general solution of the differential equation $\frac{dy}{dx}+\frac{y^2+y+1}{x^2+x+1}=0$ is given by (x+y+1)=A(1-x-y-2xy), where A is parameter. (S)

81. Find the equation of the curve passing through the point $\left(0,\frac{\pi}{4}\right)$ whose differential equation is sinxcosydx+cosxsinydy=0. (S)

82. Find the particular solution of the differential equaton: $(1+e^{2x})dy+(1+y^2)e^x dx=0$ given that y=1 when x=0. (S)

83. Solve the differential equation: $ye^{\frac{x}{y}}dx=(xe^{\frac{x}{y}}+y^2)dy(y\neq 0)$. (S)

84. Find a particular solution of the differential equation (x-y)(dx+dy)=dx-dy, given that y=-1, when x=0. (S)

85. Solve the differential equation : $\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right]\frac{dx}{dy}=1(x\neq 0)$. (S)

86. Find a particular solution of the differential equation :
$\frac{dy}{dx}+y\cot x=4x\cos ecx(x\neq 0)$, given that y=0 when x=$\frac{\pi}{2}$. (S)

87. Find a particular solution of the differential equation $(x+1)\frac{dy}{dx}=2e^{-y}-1$, given that y=0 when x=0. (S)

88. The population of a village increases continuously at the rate proportional to the number of its inhabitants present at any time. If the population of the village was 20,000 in 1999 and 25000 in the year 2004, what will be the population of the village in 2009? (S)

89. Find the general solution of the differential equation: $\frac{ydx-xdy}{y}=0$. (A)

90. Find the general solution of the differential equation: $e^x dy+(ye^x+2x)dx=0$. (A)

CHAPTER-10
Vector algebra

One mark problem

1.Find the magnitude of the vector $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$. [K]

2.Find the magnitude of the vector $\vec{a} = 2\hat{\imath} - 7\hat{\jmath} - 3\hat{k}$. [K]

3.Find the magnitude of the vector $\vec{a} = \frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} - \frac{1}{\sqrt{3}}\hat{k}$. [K]

4.Define collinear vectors. [K]

5.Define negative of a vector. [K]

6.Define a unit vector. [K]

7.Define a null vector. [K]

8.Define cointial vector. [K]

9.Define position vector of a point. [K]

10.When the two vectors are said to be equal. [K]

11.Write two different vectors having same magnitude. [U]

12.Write two different vectors having same direction. [U]

13.Find the values of x and y so that the vectors $2\hat{\imath} + 3\hat{\jmath}$ and $x\hat{\imath} + y\hat{\jmath}$ are equal. [K]

14.Find the unit vector in the direction of the $\vec{a} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$ [U]

15.Find the unit vector in the direction of the $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ [U]

16.If $\vec{a}$ is a non zero vector of magnitude a and $\lambda\vec{a}$ is a unit vector ,find the value of λ. [K]

17.For what value of λ, the vectors $\vec{a} = 2\hat{\imath} - 3\lambda\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$ are perpendicular to each other? [U]

18.For what value of λ, is the vector $\frac{2}{3}\hat{\imath} - \lambda\hat{\jmath} + \frac{2}{3}\hat{k}$ a unit vector? [U]

19. Find the value of x for which $x(\hat{\imath} + \hat{\jmath} + \hat{k})$ is a unit vector. [U]

20.Show that the vectors $2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $-4\hat{\imath} + 6\hat{\jmath} - 8\hat{k}$ are collinear. [U]

21.If $2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $a\hat{\imath} + 6\hat{\jmath} - 8\hat{k}$ are collinear then find a. [U]

22. Write the vector joining the points A(2,3,0) and B(-1,-2,-4). [U]

23. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitude $\sqrt{3}$ and 2 respectively and $\vec{a}\,.\,\vec{b} = \sqrt{6}$. [U]

24.If $\vec{a} = x\hat{\imath} + 2\hat{\jmath} - z\hat{k}$ and $\vec{b} = 3\hat{\imath} - y\hat{\jmath} + \hat{k}$ are two equal vectors,then write the value of x+y+z.[U]

25.Find the direction cosines of the vector $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$. [U]

26.Write the scalar components of the vector joining the points $A = (x_1, y_1, z_1)$ and

$B = (x_2, y_2, z_2)$. [K]

27. If vector $\overrightarrow{AB} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\overrightarrow{OB} = 3\hat{\imath} - 4\hat{\jmath} + 4\hat{k}$, find the position vector $\overrightarrow{OA}$. [U]

28. Find the scalar components of vector with initial point $(2,1)$ and terminal point $(-7,5)$.

29.Find the unit vector in the direction of $\vec{a}+\vec{b}$, where $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$. [U]

30.Find the position vector of a point *R* which divides the line joining two points P and Q whose position vectors are $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively, in the ratio 2 : 1 internally. [U]

31.Find the position vector of the mid point of the vector joining the points P (2, 3, 4) and Q (4, 1, –2). [U]

32.Show that $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$ [U]

33.Show that the vector $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the axes OX , OY and OZ. [U]

Two mark problems:

1.Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$. [U]

2.Find angle between the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$. [U]

3.If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then show that the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular. [U]

4.Find the magnitude of two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude and such that the angle between them is 60° and their scalar product is $\frac{1}{2}$. [U]

5.Find $|\vec{x}|$, if for a unit vector $\vec{a}$, $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 12$ [U]

6. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, prove that $\vec{a}$ and $\vec{b}$, are perpendicular. [U]

7. Find $|\vec{b}|$, if $(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 8$ and $|\vec{a}| = 8|\vec{b}|$. [U]

8.Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$. [U]

9.Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. [U]

10.If two vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a}.\vec{b} = 4$, find $|\vec{a} - \vec{b}|$. [U]

11.If $\vec{a} = 4\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{k}$, then find $|2\vec{b} \times \vec{a}|$ [U]

12.Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$. [U]

13.Find the area of a parallelogram whose adjacent sides are given by the vectors $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.[U]

14.Find the area of the parallelogram whose adjacent sides determine by the vectors $\vec{a}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$.[U]

15.Find the area of the triangle whose adjacent sides are determined by the vectors $\vec{a}=-2\hat{i}-5\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}-\hat{k}$.[U]

16.Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}|=3$, $|\vec{b}|=\frac{\sqrt{2}}{3}$ and $\vec{a}\times\vec{b}$ is a unit vector.Find the angle between $\vec{a}$ and $\vec{b}$
[U]

17.Find λ and μ, if $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+\lambda\hat{j}+\mu\hat{k})=\vec{0}$.
[U]

18.If $\vec{a}.\vec{a}$ =0 and $\vec{a}.\vec{b}$ =0, then what can be concluded about the vector $\vec{b}$?
[U]

19. Show that the points A(-2,3,5), B(1,2,3) and C(7,0, -1) are collinear.
[U]

20. Show that the points A(1,2,7), B(2,6,3) and C(3,10, -1) are collinear.
[U]

21.Show that $|\vec{a}|\,\vec{b}+|\vec{b}|\,\vec{a}$ is perpendicular to $|\vec{a}|\,\vec{b}-|\vec{b}|\,\vec{a}$,for any two nonzero vectors $\vec{a}$ and $\vec{b}$.
[U]

22.If either vector $\vec{a}$ =0 or $\vec{b}$ =0 ,then $\vec{a}.\vec{b}$ =0.But the converse need not be true.Justify your answer with an example.
[U]

23.Find $|\vec{a}\times\vec{b}|$, if $\vec{a}=\hat{i}-7\hat{j}+7\hat{k}$ and $\vec{b}=3\hat{i}-2\hat{j}+2\hat{k}$
[U]

24.Find the area of the triangle with vertices A(1,1,2), B(2,3,5) and C(1,5, 5)
[U]

25.The two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$ .Find the unit vector parallel to its diagonal.
[U]

26.Find the volume of the parallelepiped whose coterminous edges are represented by the vectors $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b}=-2\hat{i}+3\hat{j}-4\hat{k}$ and $\vec{c}=\hat{i}-3\hat{j}+5\hat{k}$.
[U]

27.Find the projection of $\vec{a}$ on $\vec{b}$ if $\vec{a}.\vec{b}$ =8 and $\vec{b}=2\hat{i}+6\hat{j}+3\hat{k}$
[U]

Three mark problems:

1.Show that the position vector of the point P, which divides the line joining the points A and B having the position vectors $\vec{a}$ and $\vec{b}$ internally in the ratio $m:n$ is $\frac{m\vec{b}+n\vec{a}}{m+n}$.
[A]

2.Find a unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$. [U]

3.Find λ, if the vectors $\vec{a}=\hat{i}+3\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}-\hat{k}$ and $\vec{c}=\lambda\hat{i}+7\hat{j}+3\hat{k}$ are coplanar.
[U]

4.Show that the four points A, B, C and D with position vectors $4\hat{i}+5\hat{j}+\hat{k}$, $-\hat{j}-\hat{k}$, $3\hat{i}+9\hat{j}+4\hat{k}$ and $4(-\hat{i}+\hat{j}+\hat{k})$ respectively are coplanar.
[U]

5.Find λ, such that the four points A (3, 2, 1), B (4, λ, 5), C (4, 2, –2) and D (6, 5, –1) are coplanar.
[U]

6.Show that the four points with position vectors $4\hat{i}+8\hat{j}+12\hat{k}$, $2\hat{i}+4\hat{j}+6\hat{k}$, $3\hat{i}+5\hat{j}+4\hat{k}$ and $5\hat{i}+8\hat{j}+5\hat{k}$ are coplanar.

[U]

7.Prove that $[\vec{a}\ \vec{b}\ \vec{c}+\vec{d}]=[\vec{a}\ \vec{b}\ \vec{c}]+[\vec{a}\ \vec{b}\ \vec{d}]$
[U]

8.Prove that $[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}]=2[\vec{a}\ \vec{b}\ \vec{c}]$
[U]

9.Show that the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar if $\vec{a}-\vec{b}$, $\vec{b}-\vec{c}$ and $\vec{c}-\vec{a}$ are coplanar.
[U]

10.If $\vec{a},\vec{b},\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, find the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$.
[U]

11.Show that the points $A(2\hat{i}-\hat{j}+\hat{k})$, $B(\hat{i}-3\hat{j}-5\hat{k})$ and $C(3\hat{i}-4\hat{j}-4\hat{k})$ are the vertices of a right angled triangle
[U]

12.If the vertices A, B and C of a triangle are $(1,2,3)$, $(-1,0,0)$ and $(0,1,2)$ respectively,then find the $\angle ABC$.[U]

13. Three vectors $\vec{a},\vec{b}\&\vec{c}$ satisfy the condition $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, . Evaluate the quantity $\mu=\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$, if $|\vec{a}|=1, |\vec{b}|=4 \ \& |\vec{c}|=2$.
[U]

14. Find the vector of magnitude 5 units and parallel to the resultant of the vectors
$\vec{a}=2\hat{\imath}+3\hat{\jmath}-\hat{k}$ and $\vec{b}=\hat{\imath}-2\hat{\jmath}+\hat{k}$
[U]

15. If $\vec{a},\vec{b}$ & $\vec{c}$ are three vectors such that $|\vec{a}|=3$, $|\vec{b}|=4$, $|\vec{c}|=5$ & each vector is orthogonal to sum of the other two vectors then find $|\vec{a}+\vec{b}+\vec{c}|$.
[U]

16.If $\hat{\imath}+\hat{\jmath}+\hat{k}$, $2\hat{\imath}+5\hat{\jmath}$, $3\hat{\imath}+2\hat{\jmath}-3\hat{k}$ & $\hat{\imath}-6\hat{\jmath}-\hat{k}$ are the position vectors of points A,B,C & D respectively then find the cosine angle between $\overrightarrow{AB}$ & $\overrightarrow{CD}$.
[U]

17.If $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$ & $\vec{c} = \hat{\imath} + 3\hat{\jmath} - \hat{k}$ such that $\vec{a}$ is perpendicular to $(\lambda\vec{b} + \vec{c})$ then find λ.
[U]

18.The two adjacent sides of a parallelogram are $2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$, $\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$ then find the unit vector parallel to its diagonal.Also find area of the parallelogram. [U]

19.The scalar product of the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ & $\lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is equal to one. Find the value of λ. [U]

20. If $\hat{\imath}, \hat{\jmath}, \hat{k}$ are mutually perpendicular unit vectors , $\vec{\alpha} = 3\hat{\imath} - \hat{\jmath}$, $\vec{\beta} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}$, then express $\vec{\beta}$ in the form $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$ where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$. [U]

21.If $\vec{a} = \hat{\imath} + \hat{\jmath} - \hat{k}$, $\vec{b} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$ & $\vec{c} = -\hat{\imath} + 2\hat{\jmath} - \hat{k}$ then find the unit vector perpendicular to both $\vec{a} + \vec{b}$ & $\vec{b} + \vec{c}$. [U]

22. If $\vec{a} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k}$, $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$ & $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$ then find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ & $\vec{b}$ and $\vec{c}.\vec{d} = 15$. [U]

23.If $\vec{a}$,$\vec{b}$ & $\vec{c}$ are mutually perpendicular vectors of equal magnitudes then prove that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}$,$\vec{b}$ & $\vec{c}$. [U]

PUE

CHAPTER-11

THREE DIMENSIONAL GEOMETRY

ONE MARK QUESTIONS:

1. If a line makes the angles 90^0 ,60^0 and 30^0 with the positive directions of x ,z and z-axes respectively, find its direction cosines. (U)

2. If a line makes angles 90^0,135^0 and 45^0 with the positive direction of x, y and z-axes respectively ,find its direction cosines. (U)

3. If a line has direction ratios 2,-1,-2 , determine its direction cosines. (U)

4. If a line has direction ratios -18,12,-4 then what are its direction ratios. (U)

5. Find the direction cosines of the line passing through two points (-2,4,-5) and (1,2,3). (U)

6. Find the direction ratios of the line $\frac{x-1}{2}$ = 3y = $\frac{2z+3}{4}$. (U)

7. The cartesian equation of a line is $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$. Write its Vector form. (U)

8. Find direction cosines of x-axis. (K)

9. Find direction cosines of y-axis. (K)

10. Find direction cosines of z-axis. (K)

11. Find the direction cosines of the line which makes equal angles with coordinate axis. (U)

12. Find the direction cosines of the line passing through two points (-2,4,-5) and (1,2,3). (U)

13. Find the distance of the plane 2x-3y+4z=5 from the origin. (U)

14. Find the distance of the plane z=2 from the origin. (U)

15. Find the direction cosines of the normal to the plane x + y + z =1.(U)

16. Find the direction cosines of the normal to the plane 5y+8 = 0.(U)

17. Find the cartesian equation of the plane $\vec{r}$.(i+j+k)=2. (U)

18. Find intercept cut off the plane 2x+y-z=5. (U)

19. Find the equation of the plane with intercept 4 on z -axiz and parallel to XOY plane. (U)

20. Find equation of the plane cuts coordinate axis at (a,0,0) ,(0,b,0),(0,0,c). (U)

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21. Find the vector equation of the plane passing through the point (1,0,-2) and normal to the plane $\vec{i}+\vec{j}-\vec{k}$. (U)

22. Find cartesian equation of the plane passing through the point (1,4,6) and normal to the vector $\vec{i}-2\vec{j}+\vec{k}$. (U)

23. Find vector equation of the plane passing through the point(1,2,3) and perpendicular to the vector $\vec{r}.\left(\vec{i}+2\vec{j}-5\vec{k}\right)-9=0$. (U)

24. Find the cartesian equation of the plane $\vec{r}.\left(2\vec{i}+3\vec{j}-4\vec{k}\right)=1$. (U)

25. Show that the planes 2x+y+3z-2=0 and x-2y+5=0 are perpendicular. (U)

26. Show that the planes 2x-y+3z-1=0 and 2x-y+3z+3=0 are parallel. (U)

27. Find the equation of the line in vector form which passes through (1,2,3) and parallel to the vector $3\vec{i}+2\vec{j}-2\vec{k}$. (U)

28. Define skew lines. (K)

29. Find the distance between the 2 planes 2x+3y+4z=4 and 4x+6y+8z=12. (U)

TWO MARKS QUESTIONS:

1. Find the equation of the line which passes through the point (1,2,3) and parallel to the vector $3\vec{i}+2\vec{j}-2\vec{k}$,both are in vector and cartesian form. (U)

2. Find the vector equation of the line passing through the points (3,-2,-5) and (3,-2,6). (U)

3. Find the vector equation of the line passing through the points (-1,0,2) and (3,4,6). (U)

4. Find the angle between the lines $\vec{r}=3\vec{i}+2\vec{j}-4\vec{k}$+λ($\vec{i}+2\vec{j}+2\vec{k}$.) and $\vec{r}=5\vec{i}-2\vec{j}+\mu\left(3\vec{i}+2\vec{j}+6\vec{k}\right)$ (U)

5. Find the angle between the lines $\frac{x+3}{3}=\frac{y-1}{5}=\frac{z+3}{4}and\frac{x+1}{1}=\frac{y-4}{1}=\frac{z-5}{2}$. (U)

6. Show that the line passes through the points (4,7,8) , (2,3,4) is parallel to the line passing through the points (-1,2,1) and (1,2,5). (U)

7. Find the vector equation of the plane which is at a distance 7 units from the origin and which is normal to the vector $3\vec{i}+5\vec{j}-6\vec{k}$. (U)

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8. Find the vector equation of the plane which is at a distance of $\frac{6}{\sqrt{29}}$ units from the origin and its normal vector from the origin is $2\vec{i} - 3\vec{j} + 4\vec{k}$. (U)

9. Find the equation of the plane passing through the line of intersection of the planes 3x-y+2z-4=0 and x+y-z-2 =0 and the point (2,2,1). (U)

10. Show that the points (2,3,4) ,(-1,-2,1) and (5,8,7) are collinear. (U)

11. Show that the line through the points (1,-1,2) ,(3,4,-2) is perpendicular to the line through the points (0,3,2) and (3,5,6). (U)

12. Find the distance of the point (3,-2,1) from the plane 2x-y+2z-3=0. (U)

13. Find the distance of the point (2,5,-7) from the plane $\vec{r}.\left(6\vec{i} - 3\vec{j} + 2\vec{k}\right) = 4$. (U)

14. Find the angle betweens the planes 3x-6y+2z=7 and 2x+2y-2z=5. (U)

15. Find the vector equation of the line passing through the points (-1,0,2) and (3,4,6). (U)

16. Find the vector and cartesian equation of the plane passing through the point (1,0,-2) and normal to the plane is i+j-k. (U)

THREE MARKS QUESTIONS:

1. Find the shortest between the lines l_1 and l_2 whose vector equations are $\vec{r} = i + j + \lambda(2i - j + k)$and $\vec{r} = 2i + j - k + \mu(3i - 5j + 2k)$. (U)

2. Find distance between the lines l_1 and l_2 given by $\vec{r} = i + 2j - 4k + \lambda(2i + 3j + 6k)$and $\vec{r} = 3i + 3j - 5k + \mu(2i + 3j + 6k)$ (U)

3. Find shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$. (U)

4. Find the value of p,so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles. (U)

5. Find the vector and cartesian equation of the line passing through the point (3,-2,-5) and (3,-2,6). (U)

6. Find the vector equation of the plane passing through the three points (1,-1,-1) ,(6,4,-5) and (-2,2,-1) . (U)

7. Find the equation of the line which passes through the point (1,2,3) and is parallel to the vector 3i+2j-2k both in vector and cartesian form. (U)

8. Find the direction cosines of unit vector perpendicular to the plane $\vec{r}.\left(6\vec{i}-3\vec{j}-2\vec{k}\right)+1=0$passing through origin. (U)

9. Find vector equation of the plane passing through the intersections of the planes

   3x-y+2z-4=0 and x+y+z+2=0 and the point (2,2,1). (U)

10. Find vector equation of the plane passing through the intersections of the planes

    x+y+z=1 and 2x+3y+4z=5 which is perpendicular to the plane x-y+z=0 . (U)

11. Find distance between the point P(6,5,9) and the plane determined by the points A(3,-1,2) ,

    B(5,2,4) and C(-1,-1,6). (U)

12. Show that the lines $\frac{x+3}{-3}=\frac{y-1}{1}=\frac{z-5}{5}$and $\frac{x+1}{-1}=\frac{y-2}{2}=\frac{z-5}{5}$are co-planar. (U)

13. Find the coordinates of foot of perpendicular drawn from the origin to the plane

    2x-3y+4z-6=0. (U)

14. Find the angle between the planes 2x+y-2z=5 and 3x-6y-2z=7 using vector method. (U)

15. Find the angle between the planes 3x-6y+2z=7 and 2x+2y-2z=5 using cartesian form. (U)

16. Find the coordinates of the point where the line through the points A(3,4,1) and B(5,1,6)

    crosses x-y plane. (U)

17. Find equation of the plane passing through the point (-1,3,2) and perpendicular to each of the planes x+2y+3z=5 and 3x+3y+z=0 . (U)

FIVE MARKS QUESTIONS:

1. Derive the equation of the line in space passing through the point and parallel to the vector both in the vector form and cartesian form. (U)

2. Derive the equation of the line in space passing through two given points both in vector and cartesian form. (U)

3. Derive the angle between two lines in vector and cartesian form. (U)

4. Derive the shortest distance between skew lines both in vector and cartesian form. (A)

5. Derive the distance between the parallel lines $\vec{r}=\overrightarrow{a1}+\lambda\left(\vec{b}\right)$and $\vec{r}=\overrightarrow{a2}+\mu\left(\vec{b}\right)$. (U)

6. Derive the equation of the plane in normal form (both in vector and cartesian form). (U)

7. Derive the equation of the plane perpendicular to the given vector and passing through a given point both in cartesian and vector form. (U)

8. Derive equation of a plane passing through three non-collinear with position vectors $\vec{a}, \vec{b}, \vec{c}$(both in vector and cartesian form). (U)

9. Derive equation of the plane passing through the intersection of two given planes both in vector and cartesian form.(U)

10. Derive the condition for the co-planarity of two lines in space both in vector and cartesian form. (U)

11. Derive the formula to find distance of a point from a plane both in vector and cartesian form. (U)

PUE

CHAPTER -12

Linear programming

One mark questions:

1) Define linear programming problem
2) Define an objective function of a LPP
3) Define a constraints of a LPP
4) Define a feasible region of a LPP
5) Define a feasible solution of a LPP
6) Define a infeasible solution of a LPP
7) In a LPP, if the feasible region is bounded, with corner points. Where does the optimum value of LPP exists.
8) Define optimal)feasible) solution of a LPP
9) If he corner points of the feahble rigion determined by the linear constrants of LPP are (0.5), (4, 3), (0, 6) then find the minimum value of the objective function Z = 200x + 500y
10) If the corner points of the feasible region determined by the linear constrants of a LPP are (0, 0) (30,0) (20, 30) and (0, 50) find the maximum value of the objective function Z = 4x + y

Six mark questions

1, Solve the following linear programming problems graphically

1) Maximize z=3x+4y subject to x + y $\leq 4, x \geq 0, y \geq 0$
2) Maximize $z = 3x + 2y$ subject to $x + 2y \leq 10$, $3x + y \leq 15, x, y$
3) Maximize and minimize : $z = 5x + 10y$ subject to $x + 2y \leq 120, x + y \geq 60, x - 2y \geq 0, x, y \geq 0$.
4) Maximize and minimize : $z = x + 2y$ subject to $x + 2y \geq 100, 2x - y \leq 0, 2x + y \leq 200, x, y \geq 0$.

PUE

1) Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs Rs 60/kg and Food Q costs Rs 80/kg. Food P contains 3 units/kg of Vitamin A and 5 units / kg of Vitamin B while food Q contains 4 units / kg of Vitamin A and 2 units / kg of vitamin B. Determine the minimum cost of the mixture.

2) One kind of cake requires 200g of flour and 25g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes.

3) A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time.

- What number of rackets and bats must be made if the factory is to work at full capacity?
- If the profit on a racket and on a bat is Rs 20 and Rs 10 respectively, find the maximum profit of the factory when it works at full capacity.

4) A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs 17.50 per package on nuts and Rs 7.00 per package on bolts. How many packages of each should be produced each day so as to maximize his profit, if he operates his machines for at the most 12 hours a day ?

5) A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture of package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of Rs 7 and screws B at a profit of Rs 10. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximize his profit? Determine the maximum profit.

PUE

6) A cottage industry manufactures pedestal lamps and wooden Shades, each requiring the use of a grinding / cutting machine and a sprayer. It takes 2 hours on grinding / cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding / cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding / cutting machine for at the most 12 hours. The profit from the sale of a lamp is Rs 5 and that from a shade is Rs 3. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximize his profit ?

7) A company manufactures two types of novelty souvenirs made of plywood. Sourvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit is Rs 5 each for type A and Rs 6 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximize the profit?

8) A merchant plants to sell two types of personal computers – a desktop model and a portable model that will cost Rs 25,000 and Rs 40,000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should slock to get manimum profit if he does not want to invest more than Rs 70 lakhs and if his profit on the desktop model is Rs 4,500 and on portable model is Rs 5000.

9) There are two types of fertilizers F_1 and F_2, F_1 consists of 10% nitrogen and 6% phosphoric acid and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs atlesast 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If F_1 costs Rs 6/kg and F_2 costs Rs 5/kg. determine how much of each type of fertilizer should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost ?

10) (Diet problem) : A dietician has to develop a special diet using two foods P & Q. each packet (containing 30g) of food P contains 12 units of calcium, 4 units of iron 6 units of cholesterol and 6 units of vitamin A. each packet of the same quantity of food Q contains Z unit of calcium, 20 units of iron, 4 units of chaleshol and Z units of vitamin A. the diet requires atleast 240 units of calcium, at least 460 units of iron and atmost 300 units of

cholesterol. How many packets of each food should be used to minimize the amount of vitamin A in the diet ?That is the minimum amount of vitamin A.

11) Manufacturing problem : A manufacturing company makes two models A and B of a product. Each piece of model A requires 9 labour hour for fabricating and 1 labour hour for funishing. Each piece of model B requires 12 labour hours for fabricating and 3 labour hour for finishing. For fabricating and finishing, the maximum labour house available are 180 and 30 respectively. The company makes a profit of Rs 8,000 on each piece of model A & Rs 12,000 on each piece of model B. how many pieces of model A & B should be manufactured in a week to realize maximum profit. What is maximum profit per week.

12) (allocation problem) : A cooperative society of farmers has 50 hectares of land to grow two crops X and Y. the profit from the crops X & Y per hectare are estimated as Rs 10,500 and Rs 9,000 respectively to control weeds, a liquid herbicide has to be used for crops X and Y at rates of 20 litres and 10 litres per hectare. Further no more than 800 litres of herbiade should be used in order to protect fish and wild life using a pond which collects rinage from this land. How much land should be allocated each crop so as to maximize the total profit of the society.

PUE

CHAPTER-13
PROBABILITY

1 mark questions

1) If $P(A) = \frac{7}{13},\ P(B) = \frac{9}{13}\ and\ P(A \cap B) = \frac{4}{13}$ then find $P(A|B)$. **(U)**
2) If $P(E) = 0.6, P(F) = 0.3\ and\ P(E \cap F) = 0.2$ then find $P(E|F)$. **(U)**
3) If $P(B) = 0.5\ and\ P(A \cap B) = 0.32$ then find $P(A|B)$. **(U)**
4) If $P(A) = \frac{1}{2}\ and\ P(B) = 0$ then find $P(A|B)$ if exists. **(K)**
5) If P(A) = 0 and P(B) = ½, then find P(A $/B$) if exists. **(U)**
6) If A and B are two events such that P(A) $\neq 0$. Find P(B/A) , if A is a sub set of B. **(U)**
7) If $2P(A) = P(B) = \frac{5}{13}\ and\ P(A|B) = \frac{2}{5}$ then find $P(A \cap B)$. **(U)**
8) If $P(A) = 0.8, P(B) = 0.5, P(B|A) = 0.4$ then find $P(A \cap B)$. **(U)**
9) If A and B are two events such that P(A) $\neq 0$. Find P(B/A) ,if $A \cap B = \emptyset$.**(K)**
10) If P(A) = 0.3 and P(B) = 0.4 find P(A $/B$), if A and B are mutually exclusive events. **(K)**
11) If $P(A) = \frac{7}{13},\ P(B) = \frac{9}{13}\ and\ P(A \cap B) = \frac{4}{13}$ then find $P(A|B)$. **(U)**
12) If $P(A) = \frac{7}{13},\ P(B) = \frac{9}{13}\ and\ P(A \cap B) = \frac{4}{13}$ then find $P\left(A^{l}|B\right)$. **(K)**
13) If P(A) = $\frac{6}{11}$ and P(B/A) = $\frac{2}{3}$, then find P(A ∩ B). **(U)**
14) If A and B are two events such that P(A) $+ P(B) - P(A \cap B) = P(A)$ .Find P(A/B) . **(U)**
15) If $P(A|B) = P(B|A)\ then\ prove\ that\ P(A) = P(B)$. **(K)**
16) If A is a subset of B then prove that $P(B|A) = 1$. **(K)**
17) If $P(A) \neq 0$, prove that $P(A|A) = 1$. $(\mathbf{K})$
18) Define independent events. $(\mathbf{K})$
19) If P(A) = 0.3 and P(B) = 0.4 find P(A $/B$), if A and B are independent events. **(U)**
20) Given that the event A and B are such that P(A)= $\frac{1}{2}$, P(A∩B) = $\frac{3}{5}$ and P(B) = k, find k if A and B are independent. **(U)**
21) If $P(A) = \frac{3}{5}, P(B) = \frac{1}{5}$ then find $P(A \cap B)$ if A & B are independent events. **(U)**
22) Let E and F be two events such that $P(E) = \frac{3}{5}, P(F) = \frac{3}{10}\ P(E \cap F) = \frac{1}{5}$. Are E and F independent events. **(U)**
23) If A and B are independent events with $P(A) = 0.3, P(B) = 0.4$ then find $P(\text{not A and not B})$. **(U)**
24) If A and B are independent events such that $P(A) = 0.3\ and\ P(B) = 0.6$, find P(A and B). **(U)**
25) If A and B are independent events such that $P(A) = 0.3\ and\ P(B) = 0.6$, find P(A and not B). **(U)**
26) If A and B are independent events with $P(A) = 0.3, P(B) = 0.4$ then find $P(A|B)$. **(U)**
27) Define Theorem of total probability**.** **(K)**
28) An urn contains 5 red and 2 black balls, two balls are randomly selected. Let x represents the number of black balls. What are the possible values of x. **(K)**
29) An urn contains 4 green and 2 black balls. Two balls are randomly selected. Let X represents the number of black balls, what are the possible values of X? **(K)**

30) If x represent the difference between the number of heads and number of tails obtained when a coin is tossed six times, , what are the possible values of X? **(K)**

31) If X denote the sum of the numbers obtained when two fair dice are rolled. What are possible values of X. **(K)**

32) A coin is tossed 3 times let x represents the number of tails. What are the possible value of x. **(K)**

33) Probability distribution of x is

X	0	1	2	3	4
P(X)	0	k	2k	2k	k

Where k is constant ,then find k. **(U)**

34) Probability distribution of x is

X	0	1	2	3	4
P(X)	0.1	k	2k	2k	k

Find k. **(U)**

35) The random variable has a probability distribution P (x) has following form where k is constant. $P(x) = \begin{cases} k & if\ x = 0 \\ 2k & if\ x = 1 \\ 3k & if\ x = 2 \\ 0 & is\ other\ wise. \end{cases}$ Find value of k. **(U)**

36) Is the given distribution of X is a Probability distribution of random variable X.

X	0	1	2	3	4
P(X)	0.1	0.5	0.2	0.1	0.3

(U)

37)Give the reason for the following distribution of X is not a Probability distribution of random variable X.

x	0	1	2	3	4
P(x_1)	0.1	0.5	0.2	-0.1	0.3

(U)

Two mark questions

1. If $2P(A) = P(B) = \frac{5}{13}$ $and\ P(A|B) = \frac{2}{5}$ then find $P(A \cup B)$. **(U)**
2. Prove that $P(A^l|B) = 1 - P(A|B)$. **(U)**
3. If A and B are independent events with $P(A) = 0.3, P(B) = 0.4$ then find $P(A \cup B)$. **(U)**
4. If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ $and\ P(A \cup B) = \frac{7}{11}$ then find $P(B|A)$. **(U)**
5. A and B are an events such that $P(A) = \frac{1}{2}$ $P(A \cup B) = \frac{3}{5}, P(B) = q,$ then find q if A and B are independent. **(U)**
6. Let A and B are two events such that $P(A) = \frac{1}{4}, P(A|B) = \frac{1}{2}$ and $P(B|A) = \frac{2}{3}$ then find $P(B) = ?$ **(U)**
7. A fair die is rolled consider an events $E = \{2,4,6\}$, $F = \{1,2\}$ then find $P(E|F)$. **(K)**

8. A fair die is rolled. Consider events E ={ 1,3,5 } and F = {2,3,5} , find P(F/E). **(K)**
9. A couple has two children. Find the probability that both children are males if it is known that at least one of the children is male. **(K)**
10. Mother, Father and son line up at random for a family picture , find P(E/F).If E: son on one end, F: father in middle. **(K)**
11. Consider an experiment of tossing two fair coins simultaneously. Find the probability that both are heads. Given that at least one of them is head. **(K)**
12. A couple has 2 children find the probability that both are female if it is known that elder child is female. **(K)**
13. Given that the 2 number appear in on throwing two dices are different. Find the probability of an event the sum of the number is 4. **(K)**
14. Find the conditional probability of obtaining the sum 8 given that the red die resulted is a number less than 4. **(K)**
15. In a hostel 60% of students read Hindi newspaper, 40% of students read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random then, If she reads Hindi newspaper find the probability that she also reads English newspapers. **(K)**
16. A coin is tossed 3 times then find $P(E|F)$, where E : Head on third toss and F : Head on first two tosses .**(U)**
17. A coin is tossed 3 times then find $P(E|F)$,where E :at least two heads and F : at most two heads.(**U)**
18. A black and red dice are rolled. Find the conditional probability of obtaining the sum greater than 9. Given that black die resulted as 5. **(U)**
19. A black and red dice are rolled. Find the conditional probability of obtaining the sum greater than 9. Given that black die resulted as 5. **(U)**
20. If A and B are independent events, then prove that A and B' are also independent. **(K)**
21. If A and B are independent events, then prove that A' and B are also independent. **(K)**
22. If A and B are independent events, then prove that A' and B' are also independent. **(K)**
23. If A and B are two independent events then prove that the probability of occurrence of at least one of A and B is given by $1 - P(A').P(B')$. **(K)**
24. If $P(A) = \frac{1}{2}, P(B) = \frac{7}{12}$ and $P(not\ A\ or\ not\ B) = \frac{1}{4}$ then state whereas A or B are independent. **(U)**
25. If A and B are two events such that $P(A) = \frac{1}{4} and\ P(B) = \frac{1}{2}$. Then $P(A \cap B) = \frac{1}{8}$ then
    Find P(not A and not B) . **(U)**
26. Find the probability of getting even prime number on each die, when a pair of dice is rolled. **(U)**
27. Two cards are drawn random without replacement from a pack of 52 playing cards. Find the probability that both are black cards. **(U)**
28. Two cards are drawn successfully with replacement from a pack 52 cards find the probability distribution of number of ace cards. **(U)**
29. A Urn contains 10 black and 5 white balls, 2 balls are drawn one after the other without replacement. What is the probability that both drawn balls are black. **(U)**
30. Three cards drawn successively without replacement from a pack of 52 well shuffled cards. What is the probability that 1^{st} two cards are king and 3^{rd} card drawn is ace. **(U)**
31. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls find the probability that both are red. **(U)**

32. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls find the probability that 1st ball is black and second is red. **(U)**
33. A die is tossed thrice. Find the probability of getting an odd number at least once. **(U)**
34. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently. Find the probability that the problem is solved. **(U)**

35. Find the probability distribution of number of heads in two tosses of a coin. **(K)**
36. . Find the probability distribution of number of tails in three tosses of a coin. **(K)**
37. Find the probability distribution number of success in 2 tosses of die where success is defined as number > 4. **(K)**
38. Probability distribution of X is

X	0	1	2	3	4	5	6	7
P(X)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Find i) k ii) P(X<3) . **(K)**

39. The probability distribution of the random variable X is given

x	0	1	2	3
P(x_1)	1/8	3/8	3/8	1/8

Find E(X). **(K)**

40. If P(E_1) = ½ , P(E_2)= ½ and P(A/E_1)= ½, P(A/E_2)= ¼ . Find P(E_1/A). **(K)**
41. If E(X) = $\frac{21}{6}$ and E$(X^2) = \frac{91}{6}$,find the standard deviation of X. **(K)**
42. If Var (X) = 9 and E$(X^2) = 25$, find E(X). **(K)**
43. The probability distribution of X is ,

X	0.5	1	1.5	2
P(X)	K	K^2	$2K^2$	K

Find the mean of X. **(K)**

Three mark questions.

1. A die is thrown twice and sum of the numbers appeared is observed to be six.
   What is the conditional probability that the number 4 has appeared at least once. **(U)**
2. A die is thrown 3 times events A and B are defined as follows. Event A : 4 on first throw and Event B : 6 and 5 on second and third throw. Find the probability of 'A' given that 'B' has already occurred. **(U)**
3. A pair of die are thrown, an event A and B are as follows, A : the sum of 2 numbers on the die is 8 and B : there is an even number on the first die. Find the conditional probability $P(B|A)$. **(U)**

4. 10 cards numbered from 1 to 10 are placed in a box mix up thoroughly and 1 card is drawn random, if it is known that the number on the drawn card is more than 3. What is the probability that it is an even number. **(U)**
5. An instructor has question bank consisting of 300 easy true/false questions, 200 difficult true/false questions, 500 easy multiple choice questions and use 400 difficult MCQ's. If a question is selected at random from the question bank. What is the probability that it will be a easy question given that it's a MCQ. **(U)**
6. One card is drawn at random from a well shuffled deck of 52 cards. Find where events E and F are independent. E : the card drawn is a spade and F: the card drawn is an ace.**(U)**
7. A die is marked 1,2,3 in red and 4,5,6 in green is tossed. Let 'A' be an event that 'the number is even' and B be an event that 'the number is red. Are A and B independent.
8. An unbiased dies is thrown twice, let A be an event 'odd number on the first thrown' let 'B' be an event odd number on the 2^{nd} thrown check the independence of the events A and B . **(U)**
9. A die is thrown, if E bean event, the number appearing is a multiple of 3 and F be an event the number appearing is even, then find whether E and F are independent. **(U)**
10. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag. What is the probability that the ball is red. **(U)**
11. An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red ? (A)
12. Bag I contains 3 red and 4 black bolls and Bag II contains 4 red and 5 black bolls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II . The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black. **(U)**
13. A bag contains 3 red and 4 black balls, another bag contains 5 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag. **(U)**
14. There are three coins, one is a two headed coin, another is a biased coin that comes up head 75% *of the time and third is* an unbiased coin. One of the three coins is chose at random and tossed it shows head. What is the probability that it was the two headed coin. **(U)**
15. Given three identical boxes I, II and III, each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in the box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold? **(U)**
16. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probabilities of an accident are 0.01, 0.03 and 0.15 respectively. One of the insured person meets with an accident. What is the probability that he is a scooter driver? **(U)**

17. A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, where as the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A? **(U)**
18. A doctor is to visit a patient from the past experience it is known that the probabilities that he will come by train, bus scooter or by other means of transportation are $\frac{3}{10},\frac{1}{5},\frac{1}{10}$ $and \frac{2}{5}$ respectively. The probability that he will be late are $\frac{1}{4},\frac{1}{3}\&\frac{1}{12}$. If he comes by train, bus scooter respectively. But he comes by the means of transport he will not be late. When he arrive, is late. What is the probability be will come by trains. **(U)**
19. Of the students in a college it is known that 60% reside in hotel and 40% are day scholar (not residing in hostel) previous year results report that 30% of the student who reside in hostel attain 'A' grade and 20% of day scholars attain 'A' grade in their annual examination. At the end of the year one student is chosen at random from college and he has 'A' grade. What is the probability that the student is a hosteller? **(U)**
20. In a factory which manufactures bolts, machines A, B and C manufacture respectively 25%, 35% and 40% of the bolts. Of their outputs, 5, 4 & 2 percent are respectively defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by the machine B? **(U)**
21. In answering a question on a multiple choice test a student either knows the answer or guesses. Let ¾ be the probability that he knows the answer and ¼ be the probability that he guesses. Assuming that a student who guesses the answer will be correct with probability ¼ .What is the probability that a student knows the answer given that he answered it correctly. **(U)**
22. A men is know to speak truth 3 out of 4 times. He throws a die and reports that it is six. Find the probability that it is actually six. **(U)**
23. Probability that a person speaks truth is $\frac{4}{5}$, A coin is tossed a person reports that head appears. Find the probability that it is actually head. **(U)**
24. Suppose that 5% of men and 0.25% of women have grey hair, A grey haired person is selected at random, what is the probability of this person being male? Assume that there are equal number of males and females. **(U)**
25. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond. **(U)**
26. A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B? **(U)**

27. Suppose that the reliability of a HIV test is specified as follows: Of people having HIV, 90% of the test detect the disease but 10% go undetected. Of people free of HIV, 99% of the test are judged HIV–ve but 1% are diagnosed as showing HIV+ve. From a large population of which only 0.1% have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV+ve. What is the probability that the person actually has HIV? **(U)**
28. Suppose a girl throws a die if she gets 5 or 6 she tosses a coin 3 times and notes the no of head . If she gets one, 2, 3 or 4 she tosses a coin once and notes whether head or tail is obtained. If she obtained exactly one head what is the probability that she got I,2,3 or 4 with a die. **(U)**
29. Suppose a girl throws a die if she gets 5 or 6 she tosses a coin 3 times and notes the no of head . If she gets one, 2, 3 or 4 she tosses a coin once and notes whether head or tail is obtained. If she obtained exactly one head what is the probability that she got I,2,3 or 4 with a die. **(U)**
30. If A,B,C and D are 4 boxes containing coloured marbles as given below. **(U)**

Box	**Marbles**		
	Red	**White**	**Black**
A	1	3	6
B	6	2	2
C	8	1	1
D	0	6	4

31. One of the boxes has been selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from box A?, box B?, box C? **(U)**
32. Two dice are thrown simultaneously. If X denotes the number of sixes. Find the mean (expectation) of X. **(U)**
33. Find the mean number of heads in 3 tosses of a fair coin. **(U)**
34. Find the variable of the number of obtained on a thrown of unbiased die. **(U)**
35. Two cards are drawn simultaneously without replacement from a pack of 52 cards. Find the mean of the number of kings. **(U)**
36. A class has 15 students whose ages are 15,17,15,14,21,17,19,20,16,18,20,17,16,19 and 20 years. One student is selected in such a manner that each has same chance of being chosen and the age X of the selected student is recorded. Find mean of X. **(U)**
37. Two cards are drawn simultaneously without replacement from a pack of 52 cards. Let X be the number of aces obtained. Find the value of E(X). **(U)**
38. Find the mean of the numbers obtained on throwing a die having writing 1 on three faces.2 on two faces and 5 on one face. **(U)**

Five mark questions.

1. A die is thrown six time. If getting an odd number is a success, what is the probability of i) exactly five success ii)at least five successes. **(A)**
2. Find the probability of getting at most two sixes and at least three sixes in six throws of a single die. **(A)**
3. If a fair coin is tossed 10 times. Find the probability of (i) exactly six heads. and (ii) at least six heads. **(A)**
4. A fair coin is tossed 8 times find the probability of a) At most five heads b) At least five heads . **(A)**
5. A pair of dice thrown 4 times. If getting a doublet is considered as success. Find the probability of two successes and at most three successes. **(A)**
6. Find the probability of getting five exactly twice in 7^{th} throws of a die. **(A)**
7. Five cards are drawn successively with replacement from a well shuffled deck of 52 playing cards. What is the probability that i) all the five cards are spades. ii) only 3 cards are spades, and iii) none is a spade. **(A)**
8. A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize at least once and exactly once. **(A)**
9. The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05.Find the probability that out of 5 such bulbs i) none will fuse after 150 days ii)not more than one fuse after 150 days. **(A)**
10. There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than 1 defective item. **(A)**
11. In an examination 20 questions of true false type are asked suppose a student tosses a fair coin to determine his answer to each question. If the coin falls head. He answers true if it false he answers false. Find the probability that he answers at least 1 questions correctly. **(A)**
12. On a multiple choice examination with 3 possible answer for each of the 5 questions. What is the probability that a candidate would getting 4 or more correct answers just by guessing. **(A)**
13. Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. Find the probability that there is at least one defective and at most one defective egg. **(A)**
14. It is known that 10% of certain articles manufacture are defective. What is the probability that in a random sample of 12 such articles 9 are defective. **(A)**
15. Suppose x has binomial distribution $B\left(6,\frac{1}{2}\right)$ show that $x = 3$ is the most likely outcome. **(A)**
16. The probability that a student is not swimmer is $\frac{1}{5}$. Then Find the probability that out of five students, four are swimmer. **(A)**

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17. If 90% of people are right handed. What is the probability that at most 6 of a random sample of 10 people are right handed. **(A)**
18. An urn contains 25 balls of which 10 balls bear a mark X and the remaining 15 bear a mark Y. A ball is drawn at random from the urn its mark noted down and it is replaced ,6 balls are drawn in this way. Find the probability that i) all will bear X mark, ii) not more than 2 will bear Y mark. **(A)**

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Design of the Question Paper
MATHEMATICS (35) CLASS : II PUC

Time: 3 hour 15 minute ; Max. Marks: 100

(*of which 15 minute for reading the question paper*).

The weightage of the distribution of marks over different dimensions of the question paper shall be as follows:

I. Weightage to Objectives:

Objective	Weightage	Marks
Knowledge	40%	60/150
Understanding	30%	45/150
Application	20%	30/150
Skill	10%	15/150

II. Weightage to level of difficulty:

Level	Weightage	Marks
Easy	35%	53/150
Average	55%	82/150
Difficult	10%	15/150

III. Weightage to content/subject units:

Chapter No.	Chapter	No. of teaching Hours	Marks
1	RELATIONS AND FUNCTIONS	11	11
2	INVERSE TRIGONOMETRIC FUNCTIONS	8	8
3	MATRICES	8	9
4	DETERMINANTS	13	12
5	CONTINUITY AND DIFFERENTIABILITY	19	20
6	APPLICATION OF DERIVATIVES	11	10
7	INTEGRALS	21	22
8	APPLICATION OF INTEGRALS	8	8
9	DIFFERENTIAL EQUATIONS	9	10
10	VECTORS	11	11
11	THREE DIMENSIONAL GEOMETRY	12	11
12	LINEAR PROGRAMMING	7	7
13	PROBABILITY	12	11
	TOTAL	**150**	**150**

IV. Pattern of the question paper:

Part	Type of questions	Number of questions to be set	Number of questions to be answered	Remarks
A	1 mark questions	10	10	Compulsory part
B	2 mark questions	14	10	-------------
C	3 mark questions	14	10	-------------
D	5 mark questions	10	6	Questions must be asked from specific set of topics as mentioned below, under section V
E	10 mark questions (Each question with two sub divisions namely a)6 mark and b)4 mark)	2	1	

V. Instructions:

Content areas to select questions for PART – D and PART – E

a) In part D

1. **Relations and functions:** Problems on **v**erification of invertibility of a function and writing its inverse.
2. **Matrices:** Problems on algebra of matrices .
3. **Determinants:** Problems on finding **s**olution to simultaneous linear equations involving three unknown quantities by matrix method.
4. **Continuity and differentiability:** Problems on second derivatives only.
5. **Application of derivatives:** Problems on derivative as a rate measurer.
6. **Integrals:** Derivations on indefinite integrals and evaluation of an indefinite integral by using the derived formula.
7. **Application of integrals:** Problems on finding the area of the bounded region by the method of integration.
8. **Differential equations:** Problems on **s**olving linear differential equations only.
9. **Three dimensional geometry:** Derivations on three dimensional geometry (both vector and Cartesian form)
10. **Probability:** Problems on Bernoulli Trials and Binomial distribution.

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b) In PART E:

i) 6 mark questions must be taken from the following content areas

- **Integrals:** Derivations on definite integrals and evaluation of a definite integral using the derived formula.
- **Linear programming**: Problems on linear programming.

ii) 4 mark question must be taken from the following content areas.

- **Continuity and differentiability**: Problems on continuous functions.
- **Determinants**: Problems on evaluation of determinants by using properties.

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SAMPLE BLUE PRINT

II PUC MATHEMATICS (35)

TIME: 3 hours 15 minute **Max. Mark: 100**

Chapter	CONTENT	Number Teaching hours	PART A	PART B	PART C	PART D	PART E		Total marks
			1 mark	2 mark	3 mark	5 mark	6 mark	4 mark	
1	RELATIONS AND FUNCTIONS	11	1	1	1	1			11
2	INVERSE TRIGONOMETRIC FUNCTIONS	8	1	2	1				8
3	MATRICES	8	1		1	1			9
4	DETERMINANTS	13	1	1		1		1	12
5	CONTINUITY AND DIFFERENTIABILITY	19	1	2	2	1		1	20
6	APPLICATION OF DAERIVATIVES	11		1	1	1			10
7	INTEGRALS	21	1	2	2	1	1		22
8	APPLICATION OF INTEGRALS	8			1	1			8
9	DIFFERENTIAL EQUATIONS	9		1	1	1			10
10	VECTOR ALGEBRA	11	1	2	2				11
11	THREE DIMENSIONAL GEOMETRY	12	1	1	1	1			11
12	LINEAR PROGRAMMING	7	1				1		7
13	PROBABILITY	12	1	1	1	1			11
	TOTAL	**150**	**10**	**14**	**14**	**10**	**2**	**2**	**150**

Model Question Paper -1

II P.U.C MATHEMATICS (35)

Time : 3 hours 15 minute **Max. Marks : 100**

Instructions :

(i) *The question paper has five parts namely A, B, C, D and E. Answer all the parts.*

(ii) *Use the graph sheet for the question on Linear programming in PART E.*

PART – A

Answer ALL the questions **$10 \times 1 = 10$**

1. A relation R on A={1,2} defined by R={(1,1),(1,2),(2,1)} is not trasitive, why?
2. Write the principal value branch of $\sec^{-1} x$.
3. Define a diagonal matrix.
4. If A is a square matrix of order 3 and $|A|$=5, then find $|\text{adj}A|$.
5. Differentiate the function $\tan\sqrt{x}$ with respect to x.
6. Evaluate $\int \operatorname{cosec}^2\left(\frac{x}{2}\right)dx$.
7. For what value of λ, is the vector $\frac{2}{3}i - \lambda j + \frac{2}{3}k$ a unit vector?
8. Find the direction ratio of the line $\frac{x-1}{2} = 3y = \frac{2z+3}{4}$.
9. Define optimal solution in linear programming problem.
10. If P(A) = 0 and P(B) = ½, then find P(A /B) if exists.

PART B

Answer any TEN questions: **$10 \times 2 = 20$**

11. Find the gof and fog if $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$.
12. Write the function $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ x$\neq 0$, in the simplest form.
13. Prove that $2\tan^{-1}\frac{1}{2} - \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$.
14. If area of the triangle with vertices $(-2,0)$, $(0,4)$ and $(0,k)$ is 4 square units, find the value of 'k' using determinants.
15. Find $\frac{dy}{dx}$, if $y + \sin y = \cos x$, where $y \neq (2n+1)\pi$.
16. If x = 2at, y = $\frac{4}{t}$. Find $\frac{dy}{dx}$.
17. Show that the function f given by $f(x) = x^3 - 6x^2 + 17x - 420$ is strictly increasing on R.

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18. Find $\int e^x \sec x(1+\tan x)\,dx$.

19. Evaluate: $\int \frac{dx}{\sin^2 x \cos^2 x}$.

20. Find the order and degree of the differential equation $\frac{d^2y}{dx^2} = \cos 3x + \sin 3x$.

21. The position vectors of two points P and Q are $\hat{i}+2\hat{j}-\hat{k}$ and $-\hat{i}+\hat{j}+\hat{k}$ respectively. Find the position vector of a point R which divides the line $\overrightarrow{PQ}$ in the ratio 2 : 1 internally.

22. Prove that $\left[\vec{a},\vec{b},\vec{c}+\vec{d}\right]=\left[\vec{a},\vec{b},\vec{c}\right]+\left[\vec{a},\vec{b},\vec{d}\right]$.

23. Find the angle between the pair of lines $\vec{r}=3\hat{i}+5\hat{j}-\hat{k}+\lambda\left(\hat{i}+\hat{j}+\hat{k}\right)$ and $\vec{r}=7\hat{i}+4\hat{k}+\mu\left(2\hat{i}+2\hat{j}+2\hat{k}\right)$.

24. If P(E_1)= ½ ,P(E_2)= ½ and P(A/E_1)= ½, P(A/E_2)= ¼ . Find P(E_1/A).

PART C

Answer any TEN questions: **10×3=30**

25. If $*: R\times R \to R$ and $\blacksquare: R\times R\to R$ defined as $a*b = |a-b|$ and $a\blacksquare b = a$ $\forall a,b \in R$, Show that $*$ is commutative but not associative and $\blacksquare$ is associative.

26. Prove that $\cos^{-1}\frac{4}{5} + \csc^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$.

27. Express $\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrices.

28. If y = $\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$, *find* $\frac{dy}{dx}$.

29. If y = $x^x + x^a + a^x + a^a$, find $\frac{dy}{dx}$.

30. Find two positive numbers x and y such that $x+y=60$ and xy^3 is maximum.

31. *Evaluate* $\int \frac{e^x(1+x)}{cos^2(xe^x)}dx$.

32. Evaluate $\int_0^3 x^2 dx$ as the limit of sum.

33. Find the area between the curves y = x and y = x^2.

34. For the differential equation $xy\frac{dy}{dx}=(x+2)(y+2)$, find the solution curve passing through the point (1,-1).

35. Find the unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a} = 3\hat{\imath}+2\hat{\jmath}+2\hat{k}$ and $\vec{b} = \hat{\imath}+2\hat{\jmath}-2\hat{k}$.

36.If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, find the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$.

37 . Find the angle between the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane $10x+2y-11z=3$.

38.Two dice are thrown simultaneously. If X denotes the number of sixes. Find the mean (expectation) of X.

PART D

Answer any SIX questions: $6\times5=30$

39. If R_+ is the set of all non-negative real numbers prove that the function $f: R_+ \to [-5, \infty)$ defined by $f(x)=9x^2+6x-5$ is invertible. Write also $f^{-1}(x)$.

40. If A=$\begin{bmatrix}0\\1\\2\end{bmatrix}$, B = $[1 \quad 5 \quad 7]$ Verify that $(AB)^{/}=A^{/}B^{/}$.

41. Solve the following system of equation by using matrix method: x + y + z = 6,y + 3z -11= 0 and x + z = 2y.

42. If $e^y(x+1)=1, \textit{show that } \frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$.

43. The volume of a cube is increasing at a rate of 9cc/sec . How fast is the surface area increasing when the length of an edge is 10 cm.

44.Find the integral of $\frac{1}{\sqrt{x^2-a^2}}$ with respect to x and hence evaluate$\int\frac{1}{\sqrt{4x^2-25}}dx$.

45. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

46. Solve the differential equation,$x\frac{dy}{dx}+2y=x^2\log x$.

47. Derive the equation of a line in space passing through two given points both in the vector and Cartesian form.

48. If a fair coin is tossed 6 times. Find the probability of (i) at least five heads and (ii) at most five heads (iii) exactly 5 heads.

PART E

Answer any ONE question: $1\times10=10$

49. (a) Prove that $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$ and evaluate $\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}$

(b) Prove that $\begin{vmatrix}1+a^2-b^2 & 2ab & -2b\\ 2ab & 1-a^2+b^2 & 2a\\ 2b & -2a & 1-a^2-b^2\end{vmatrix}=(1+a^2+b^2)^3$

50. (a)Solve the following linear programming problem graphically: Minimize and maximize $Z=x+2y$, subject to constraints $x+2y\geq100, 2x-y\leq0,\ 2x+y\leq200,\ x,\ y\geq0$

(b) Discuss the continuity of the function $f(x)=\begin{cases}-2 & \text{if } x\leq-1\\ 2x, & \text{if} -1<x<1.\\ 2, & \text{if } x\geq1\end{cases}$

PUE

MODEL QUESTION PAPER – 2

II P.U.C MATHEMATICS (35)

Time : 3 hours 15 minute **Max. Marks : 100**

Instructions :

(i) *The question paper has five parts namely A, B, C, D and E. Answer all the parts.*

(ii) *Use the graph sheet for the question on Linear programming in PART E.*

PART – A

Answer ALL the questions **$10 \times 1=10$**

1. Define a binary operation on a set.
2. Write the range of $f(x)=\sin^{-1}x$ in $[0,2\pi]$other than $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.
3. If a matrix has 7 elements, write all possible orders it can have.
4. If A is a square matrix of order 3 and $|A|=4$, then find $|adj\ A|$.
5. If $y=e^{\log x}$, Show that $\frac{dy}{dx}=1$.
6. Evaluate $\int\left(\frac{d}{dx}\,e^{5x}\right)dx$.
7. If $\vec{a}$ is a unit vector such that $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=8$, find $|\vec{x}|$.
8. Find the equation of plane with the intercepts 2, 3 and 4 on x, y and z axis respectively
9. Define Optimal Solution in Linear Programming Problem.
10. A fair die is rolled . Consider the events E={1,3,5} and F={2,3} , find P(E|F).

.

PART B

Answer any TEN questions: **$10 \times 2=20$**

11. Define an equivalence relation and give an example.
12. Prove that $3\sin^{-1}x=\sin^{-1}(3x-4x^3),\ x\in\left[\frac{-1}{2},\frac{1}{2}\right]$.
13. Write in the simplest form of $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$, $0<x<\pi$.
14. If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, show that $A^2-5A+7I=O$. Hence find A^{-1}.

15 Prove that the function f given by $f(x)=|x-1|, x\in R$ is not differentiable at $x=1$.

16. Find 'c' of the mean value theorem for the function $f(x)=2x^2-10x+29$ in $[2,7]$.

17. Find a point on the curve $y=(x-2)^2$ at which the tangent is parallel to the x-axis.

18. Evaluate $\int \frac{\sin^2 x}{1+\cos x}\,dx$.

19. Find $\int_2^3 \frac{x}{x^2+1}\,dx$.

20. Form the differential equation of the family of curve $y^2 = a(b^2-x^2)$.

21. Find the unit vector in the direction of $\vec{a}=\hat{i}-2\hat{j}$, also find the vector whose magnitude is 7 units and in the direction $\vec{a}$.

22. If $\vec{a}\cdot\vec{b}=\sqrt{15}, |\vec{b}|=\sqrt{3}$ and angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$, find $|\vec{a}|$

23. Find the angle between the pair of planes $7x+5y+6z+30=0$ and $3x-y-10z+4=0$

24. Find the probability distribution of number of heads in two tosses of a coin.

PART C

Answer any TEN questions: $10\times 3=30$

25. If $f: N\rightarrow N$, $g: N\rightarrow N$ and $h: N\rightarrow R$ defined as $f(x)=2x$, $g(y)=3y+4$ and $h(x)=\sin x$, $\forall x,y,z$ in N. Show that $h\circ(g\circ f)=(h\circ g)\circ f$.

26. Show that $\sin^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=\pi$

27. If $A=\begin{bmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{bmatrix}$, show that $A^2=\begin{bmatrix}\cos 2x & \sin 2x\\ -\sin 2x & \cos 2x\end{bmatrix}$ and $A'A=I$.

28. Find $\frac{dy}{dx}$, if $x^3+x^2y+xy^2+y^3=81$

29. Differentiate $(x\cos x)^x+(x\sin x)^{\frac{1}{x}}$ with respect to x.

30. Find the absolute maximum value and the absolute minimum value of the function $f(x)=\sin x+\cos x$, $x\in[0,\pi]$.

31. Find $\int \frac{xe^{x}}{(1+x)^{2}}dx$

32. Evaluate: $\int \frac{2}{(1-x)(1+x^2)}\,dx$.

33. Find the area bounded by the curve $y = \cos x$ between $x = 0$ and $x = 2\pi$.

34. Find the general solution of $\frac{dy}{dx}$+y =1 (y$\neq$ 1).

35. Find the value of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ $and\,\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles.

36. Find the area of the rectangle having vertices A, B, C and D with P. V $-\hat{\imath} + \frac{1}{2}\hat{\jmath} + 4\hat{k}$, $\hat{\imath} + \frac{1}{2}\hat{\jmath} + 4\hat{k}$, $\hat{\imath} - \frac{1}{2}\hat{\jmath} + 4\hat{k}$ and $-\hat{\imath} - \frac{1}{2}\hat{\jmath} + 4\hat{k}$ respectively.

37. Show tha the four points A, B,C and D with position vectors 4i+5j+k, -(j+k), 3i+9j+4k and -4i+4j+4k respectively coplanar.

38. In answering a question on a multiple choice test a student either knows the answer or guesses. Let ¾ be the probability that he knows the answer and ¼ be the probability that he guesses. Assuming that a student who guesses the answer will be correct with probability ¼ . What is the probability that a student knows the answer given that he answered it correctly.

PART D

Answer any SIX questions: **6 × 5=30**

39. If $f : A \to A$ defined by $f(x) = \frac{4x+3}{6x-4}$, where $A = R - \left\{\frac{2}{3}\right\}$,show that f is invertible and $f^{-1} = f$.

40. *If* A=$\begin{pmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 3 & -1 & 2 \end{pmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ 0 & 2 \\ -1 & 4 \end{bmatrix}$ and C=$\begin{bmatrix} 1 & 2 & 3 & -4 \\ 2 & 0 & -2 & 1 \end{bmatrix}$ Prove that (AB)C= A(BC)

41. Solve by matrix method:

$4x + 3y + 2z = 60$, $2x + 4y + 6z = 90$, $6x + 2y + 3z = 70$.

42. If $y = 3\cos(\log x) + 4\sin(\log x)$ show that $x^2y_2 + xy_1 + y = 0$.

43. A ladder $5\,m$ long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall at the rate of $2\,m/sec$. How fast is its height on the wall decreasing when the foot of the ladder

is $4\,m$ away from the wall?.

44. Find the integral of $\sqrt{x^2 - a^2}$ with respect to x and hence evaluate

$\int \sqrt{x^2 + 4x - 5}$ dx

45. Find the area of the smaller region enclosed by the circle $x^2+y^2=4$ and the line x+y=2 by integration method.

46. Solve the differential equation $\frac{dy}{dx} + 2y\,tanx$ =sinx, y=0 when x=$\frac{\pi}{3}$

47. Derive the equation of a plane perpendicular to a given vector and passing through a given point in both vector and Cartesian form.

48. In an examination 20 question of true-false are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, the answers 'true', if it falls tails, he answers. "false" find the probabilities that he answers at least 12 questions correctly.

PART E

Answer any ONE question: **1×10=10**

49. (a) One kind of cake requires 200 gm of flour and 25 g of fat and another kind of cake requires 100 gm of flour and 50 gm of fat. Find the maximum number of cakes which can be made from 5 kg and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes.

(b) Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

50. (a) Prove that $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if f(x) is an even function} \\ 0, & \text{if f(x) is an odd function} \end{cases}$

and hence evaluate $\int_{-\pi/2}^{\pi/2} (x^3 + x\,cosx + tan^5 x + 1)dx$

(b) Find all points of discontinuity of f, where f is defined by

$f(x) = \begin{cases} 2x+3, & \text{if } x \le 2 \\ 2x-3, & \text{if } x > 2 \end{cases}$.

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MODEL QUESTION PAPER – 3

II P.U.C MATHEMATICS (35)

Time : 3 hours 15 minute **Max. Marks : 100**

Instructions :

(i) The question paper has five parts namely A, B, C, D and E. Answer all the parts.

(ii) Use the graph sheet for the question on Linear programming in PART E.

PART – A

Answer ALL the questions **10×1=10**

1. Let * be a binary operation on N given by a*b=LCM of a and b. Find 20*16.
2. What is th reflection of the graph of the function y=sinx along the line y=x.
3. What is the number of possible square matrices of order 3 with each entry 0 or 1?
4. For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ are singlular.
5. Write the derivative of $\sin^{-1}(\cos x)$ with respect to x.
6. Evaluate $\int_2^3 \frac{1}{x}dx$
7. Find λ if the vector $\hat{\imath} - \lambda\hat{\jmath} + 2\hat{k}$ and $2\hat{\imath} + 4\hat{\jmath}$ are perpendicular to each other.
8. Write the vector form of the equation of the line $\frac{x-3}{3} = \frac{y+4}{7} = \frac{z-6}{2}$
9. Define optimal solution in Linear programming problem.
10. If P(A)=0.3 , P(not B) = 0.4 and A and B are independent events , find P (A and not B).

PART B

Answer any TEN questions **10×2=20**

11. Show that the signum function f: R→R given by $f(x)=\begin{cases} 1 \ if \ x>0 \\ 0 \ if \ x=0 \\ -1 \ if \ x<0 \end{cases}$ is neither one-one nor onto.
12. Find the value of $\tan[\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}]$.
13. If the matrix A=$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and A^2=kA, then write the value of k.

14. If x=$\sqrt{a^{\sin^{-1}t}}$, y=$\sqrt{a^{\cos^{-1}t}}$, then Show that $\frac{dy}{dx} = -\frac{y}{x}$

15. Write the inverse trigonometric function $\tan^{-1}\left(\frac{1}{\sqrt{a^2-x^2}}\right)$ |x|<a, in simplest form.

16. If $\sin^2 x + \cos^2 y=1$, Show that $\frac{dy}{dx} = \frac{sin2x}{sin2y}$

17. If the radius of a sphere is measured as 9 cm with an error of 0.03 m, find the approximate error in calculating its surface area.

18. Evaluate $\int \sin^{-1}(cosx)dx$

19. Find $\int \frac{1}{\sqrt{7-6x-x^2}}dx$

20. Verify that y=$\sqrt{\boldsymbol{a^2-x^2}}$ is a solution of the differential equation $x + y\frac{dy}{dx}$=0

21. Find the magnitude of two vector $\vec{a}$ and $\vec{b}$ and their scalar product is $\frac{1}{2}$.

22. Show that $(\vec{a}-\vec{b}) \times (\vec{a}+\vec{b}) = 2(\vec{a}\times\vec{b})$

23. If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-3}$ are perpendicular, find the value of k.

24. A die is tossed thrice. Find the probability of getting an odd number at least once.

PART C

Answer any TEN questions **10 × 3=30**

25. Show that the relation R in the set A=$\{x \in Z: 0 \le x \le 12\}$ is given by R={(a,b):|a-b| is a multiple of 4} is an equivalence relation.

26. Prove that $\tan^{-1}\left(\frac{cosx}{1+sinx}\right) = \frac{\pi}{4} - \frac{x}{2}$ $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

27. Using elementary operations ,find the inverse of the matrix$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

28. If $y=(\log x)^{\cos x} + x^{\sin x}$ find $\frac{dy}{dx}$

29. If x=$a(cos\theta + \log\tan\left(\frac{\theta}{2}\right)$ and y= $a\, sin\theta$, Show that $\frac{dy}{dx} = tan\theta$.

30. At what points, the function f(x)=sinx-cosx, $0 < x < 2\pi$, attains local maxima and minima

31. Evaluate $\int e^x sinx\, dx$

32. Find $\int \frac{sinx}{sinx(x+a)}$dx

33. Find the area of the region enclosed by the circle $x^2+y^2=a^2$ by integration method.

34. Solve the differential equation $\frac{dy}{dx} = 1 - 2\frac{y^2}{x^2} + \frac{y}{x}$

35. Show that the points $A(2\hat{\imath} - \hat{\jmath} + \hat{k}), B(\hat{\imath} - 3\hat{\jmath} - 5\hat{k})$ and $C(3\hat{\imath} - 4\hat{\jmath} - 4\hat{k})$ are the vertices of right angled triangle.

36. Three vectors satisfy the condition $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{O}$, Find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$, if $|\vec{a}| = 1, |\vec{b}| = 4$ and $|\vec{c}| = 2$

37. Find the equation of the plane through the line of intersection of the planes x+y+z=1 and 2x+3y+4z=5 which is perpendicular to the plane x-y+z=0.

38. A random variable x has the following probability distribution

X	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	K^2	$2k^2$	$7k^2 + k$

Determine (i) k (ii) p (x < 3)

PART D

Answer any SIX questions **6×5=30**

39. Let R_+ be the set of all non negative real numbers , Show that the function $f:R_+ \rightarrow [4,\infty)$ defined by $f(x)=x^2 +4$ is invertible. Also find the inverse of f(x)

40. If $A = \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -3 \\ -3 & 4 \end{bmatrix}$ verify that $AB - BA$ is a skew symmetric matrix and $AB + BA$ is a symmetric matrix.

41. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations

$$2x - 3y + 5z = 11;\ 3x + 2y - 4z = -5 \text{ and } x + y - 2z = -3$$

42. If $y = e^{a\cos^{-1}x}, -1 \le x \le 1$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0.$.

43.Sand is pouring from a pipe at the rate of $12\ cm^3/sec$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is $4\ cm$?

44.Find the integral of $\frac{1}{\sqrt{x^2+a^2}}$ with respect to x and evaluate $\int \frac{1}{\sqrt{x^2+2x+2}}\,dx$.

45. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$

46. Derive an equation of a plane in the normal form both in vector and Cartesian form.

47. Solve $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{4x^2}{1+x^2}$ given that y=0 when x=0

48. The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs i) none ii) not more than one iii) more than one iv) at least once will fuse after 150 days of use?

PART E

Answer any ONE question **$1 \times 10 = 10$**

49. (a)Prove that $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ and hence evaluate

$\int_{-1}^{2} |x^3 - x|dx$.

(b) Show that $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$

50. (a) A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires atleast 240 units of calcium, atleast 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to minimise the amount of vitamin A in the diet? What is the minimum amount of vitamin A?

(b) Find the relationship between a and b so that the function f defined by $f(x) = \begin{cases} ax+1, & \text{if } x \le 3 \\ bx+3, & \text{if } x > 3 \end{cases}$ is continuous at $x = 3$.

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