This question paper contains **4** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8648**

Unique Paper Code : **62351101** **J**

Name of the Paper : **Calculus**

Name of the Course : **B.A. (Programme) Mathematics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

Marks are indicated against each question.

1. (*a*) Show that : 6

$$\lim_{x \to 0} \frac{xe^{1/x}}{1+e^{1/x}} = 0.$$

(*b*) Examine the continuity of the function :

$$f(x) = \begin{cases} 0, & \text{if} \quad x = 0 \text{ or } \frac{1}{2} \\ \frac{1}{2} - x, & \text{if} \quad 0 < x < \frac{1}{2} \\ 4x^2 - 1, & \text{if} \quad \frac{1}{2} < x < \frac{3}{4} \\ 1 - x^2, & \text{if} \quad \frac{3}{4} \le x \le 1 \end{cases}$$

at $x = 0, \frac{1}{2}, \frac{3}{4}$ and 1. Classify the type of discontinuity, wherever it exists. 6

(c) Discuss the derivability of the function :

$$f(x) = \begin{cases} 2x-3, & \text{if } 0 \le x \le 2 \\ x^2-3 & \text{if } 2 < x \le 4 \end{cases}$$

at $x = 2$ and $x = 4$. 6

2. (a) If $x = \sin t$, $y = \sin pt$; prove that : 6.5

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2 y = 0 .$$

(b) If $y = \sin^{-1} x$, show that : 6.5

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0 .$$

(c) If $u = \tan^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, use Euler's Theorem to show that : 6.5

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{4}\sin 2u .$$

3. (a) If the tangent to the curve :

$$\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1$$

cuts off intercept p and q from the axes of x and y respectively. Show that : 6

$$\frac{p}{a} + \frac{q}{b} = 1 .$$

(*b*) Find the condition for the curves :

$$ax^2 + by^2 = 1,\ a'x^2 + b'y^2 = 1$$

to intersect orthogonally. 6

(*c*) Show that the radius of curvature at a point of the curve :

$$x = ae^{\theta}(\sin\theta - \cos\theta), \quad y = ae^{\theta}(\sin\theta + \cos\theta)$$

is twice the distance of the tangent at the point from origin. 6

4\. (*a*) Find the asymptotes of the curve : 6.5

$$x^3 + x^2y - xy^2 - y^3 + 2xy + 2y^2 - 3x + y = 0.$$

(*b*) Find the position and nature of the double points on the curve : 6.5

$$x^4 + y^3 - 2x^3 + 3y^2 - a^4 = 0.$$

(*c*) Trace the curve : 6.5

$$x^2(x^2 + y^2) = 4(x^2 - y^2).$$

5\. (*a*) Verify Lagrange's mean value theorem for the function $f(x) = \log x$ in $[1,\ e]$. 6

(*b*) State Rolle's theorem and give its geometrical interpretation. Explain why Rolle's theorem is not applicable for the following function : 6

$$f(x) = \begin{cases} 1, \text{ if } x = 0 \\ x, 0 < x \le 1 \end{cases}$$

(*c*) Show that :

$$\frac{v-u}{1+v^2} < \tan^{-1} v - \tan^{-1} u < \frac{v-u}{1+u^2}, \text{ if } 0 < u < v,$$

and deduce that :

$$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}.$$ 6

6. (*a*) State and prove Cauchy's mean value theorem. 6.5

(*b*) Evaluate $\lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{e^x - 1}\right)$. 6.5

(*c*) Find the maxima and minima of the function :

$$f(x) = \sin x + \frac{1}{2}\sin 2x + \frac{1}{2}\sin 3x$$

for all $x \in [0, \pi]$. 6.5